



TECHNISCHE
UNIVERSITÄT
DRESDEN

COMPLEXITY THEORY

Lecture 13: Space Hierarchy and Gaps

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Knowledge-Based Systems

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Review

Review: Time Hierarchy Theorems

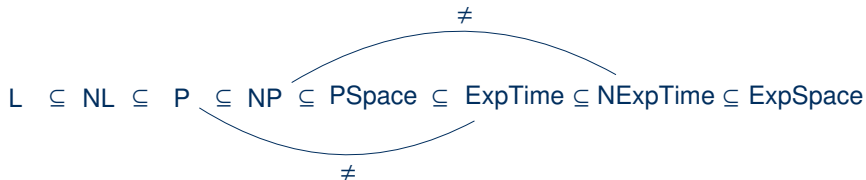
Time Hierarchy Theorem 12.12 If $f, g : \mathbb{N} \rightarrow \mathbb{N}$ are such that f is time-constructible, and $g \cdot \log g \in o(f)$, then

$$\text{DTime}_*(g) \subsetneq \text{DTime}_*(f)$$

Nondeterministic Time Hierarchy Theorem 12.14 If $f, g : \mathbb{N} \rightarrow \mathbb{N}$ are such that f is time-constructible, and $g(n+1) \in o(f(n))$, then

$$\text{NTime}_*(g) \subsetneq \text{NTime}_*(f)$$

In particular, we find that $P \neq \text{ExpTime}$ and $NP \neq \text{NExpTime}$:



A Hierarchy for Space

Space Hierarchy

For space, we can always assume a single working tape:

- Tape reduction leads to a constant-factor increase in space
- Constant factors can be eliminated by space compression

Therefore, $D\text{Space}_k(f) = D\text{Space}_1(f)$.

Space turns out to be easier to separate – we get:

Space Hierarchy Theorem 13.1: If $f, g : \mathbb{N} \rightarrow \mathbb{N}$ are such that f is space-constructible, and $g \in o(f)$, then

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Challenge: TMs can run forever even within bounded space.

Proving the Space Hierarchy Theorem (1)

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Proof: Again, we construct a diagonalisation machine \mathcal{D} . We define a multi-tape TM \mathcal{D} for inputs of the form $\langle \mathcal{M}, w \rangle$ (other cases do not matter), assuming that $|\langle \mathcal{M}, w \rangle| = n$

- Compute $f(n)$ in unary to mark the available space on the working tape
- Initialise a separate countdown tape with the largest binary number that can be written in $f(n)$ space
- Simulate \mathcal{M} on $\langle \mathcal{M}, w \rangle$, making sure that only previously marked tape cells are used
- Time-bound the simulation using the content of the countdown tape by decrementing the counter in each simulated step
- If \mathcal{M} rejects (in this space bound) or if the time bound is reached without \mathcal{M} halting, then accept; otherwise, if \mathcal{M} accepts or uses unmarked space, reject

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$L(\mathcal{D}) \in DSpace(f)$:

- f is space-constructible, so both the marking of tape symbols and the initialisation of the counter are possible in $DSpace(f)$
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There is w such that $\langle \mathcal{M}, w \rangle \in \mathbf{L}(\mathcal{D})$ iff $\langle \mathcal{M}, w \rangle \notin \mathbf{L}(\mathcal{M})$:

- As for time, we argue that some w is long enough to ensure that f is sufficiently larger than g , so \mathcal{D} 's simulation can finish.
- The countdown measures $2^{f(n)}$ steps. The number of possible distinct configurations of \mathcal{M} on w is $|Q| \cdot n \cdot g(n) \cdot |\Gamma|^{g(n)} \in 2^{O(g(n)+\log n)}$, and due to $f(n) \geq \log n$ and $g \in o(f)$, this number is smaller than $2^{f(n)}$ for large enough n .
- If \mathcal{M} has d tape symbols, then \mathcal{D} can encode each in $\log d$ space, and due to \mathcal{M} 's space bound \mathcal{D} 's simulation needs at most $\log d \cdot g(n) \in o(f(n))$ cells.

Therefore, there is w for which \mathcal{D} simulates \mathcal{M} long enough to obtain (and flip) its output, or to detect that it is not terminating (and to accept, flipping again). □

Space Hierarchies

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Corollary 13.2: $\text{PSPACE} \subseteq \text{ExpSpace}$

Proof: As for time, but easier.

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Corollary 13.4: For all real numbers $0 < a < b$, we have $\text{DSpace}(n^a) \subsetneq \text{DSpace}(n^b)$.

In other words: The hierarchy of distinct space classes is very fine-grained.

The Gap Theorem

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Yes. The following theorem shows why (for time):

Special Gap Theorem 13.5: There is a computable function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $\text{DTime}(f(n)) = \text{DTime}(2^{f(n)})$.

This has been shown independently by Boris Trakhtenbrot (1964) and Allan Borodin (1972).

Reminder: For this we continue to use the strict definition of $\text{DTime}(f)$ where no constant factors are included (no hidden $O(f)$). This simplifies proofs; the factors are easy to add back.

Proving the Gap Theorem

Special Gap Theorem 13.8: There is a computable function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $\text{DTime}(f(n)) = \text{DTime}(2^{f(n)})$.

Proof idea: We divide time into exponentially long intervals of the form:

$$[0, n], \quad [n + 1, 2^n], \quad [2^n + 1, 2^{2^n}], \quad [2^{2^n} + 1, 2^{2^{2^n}}], \quad \dots$$

(for some appropriate starting value n)

We are looking for **gaps of time** where no TM halts, since:

- for every finite set of TMs,
- and every finite set of inputs to these TMs,
- there is some interval of the above form $[m + 1, 2^m]$

such none of the TMs halts in between $m + 1$ and 2^m steps on any of the inputs.

The task of f is to find the start m of such a gap for a suitable set of TMs and words

Gaps in Time

We consider an (effectively computable) enumeration of all Turing machines:

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Definition 13.6: For arbitrary numbers $i, a, b \geq 0$ with $a \leq b$, we say that $\text{Gap}_i(a, b)$ is true if:

- Given any TM \mathcal{M}_j with $0 \leq j \leq i$,
- and any input string w for \mathcal{M}_j of length $|w| = i$,

\mathcal{M}_j on input w will halt in less than a steps, in more than b steps, or not at all.

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Lemma 13.7: Given $i, a, b \geq 0$ with $a \leq b$, it is decidable if $\text{Gap}_i(a, b)$ holds.

Proof: We just need to ensure that none of the finitely many TMs $\mathcal{M}_0, \dots, \mathcal{M}_i$ will halt after a to b steps on any of the finitely many inputs of length i . This can be checked by simulating TM runs for at most b steps. □

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$$\text{in}(n) = |\Sigma_0|^n + \dots + |\Sigma_n|^n \quad \text{where } \Sigma_i \text{ is the input alphabet of } \mathcal{M}_i$$

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We recursively define a **series of numbers** k_0, k_1, k_2, \dots by setting $k_0 = 2n$ and $k_{i+1} = 2^{k_i}$ for $i \geq 0$, and we consider the following **list of intervals**:

$$\begin{array}{ccccccc} [k_0 + 1, k_1], & [k_1 + 1, k_2], & \dots, & [k_{\text{in}(n)} + 1, k_{\text{in}(n)+1}] \\ \parallel & \parallel & & \parallel \\ [2n + 1, 2^{2n}], & [2^{2n} + 1, 2^{2^{2n}}], & \dots, & [2^{\dots^{2n}} + 1, 2^{\dots^{2n}}] \end{array}$$

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Let $f(n)$ be the least number k_i with $0 \leq i \leq \text{in}(n)$ such that $\text{Gap}_n(k_i + 1, k_{i+1})$ is true.

Properties of f

We first establish some basic properties of our definition of f :

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Claim: The function f is computable.

Proof: We can compute $\text{in}(n)$ and k_i for any i , and we can decide $\text{Gap}_n(k_i + 1, k_{i+1})$. \square

Papadimitriou: “notice the fantastically fast growth, as well as the decidedly unnatural definition of this function.”

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Therefore we have $\mathbf{L} \in \text{DTime}(f(n))$.

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Really?

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Could we still do a state-space extension as Papadimitriou suggested?

- It seems possible to do a multiplication of states to do the finite automaton detection of words $|w| < j$ alongside the normal operation of the TM
- However, we'll have to leave the movement of the heads to the original TM
- Due to the other requirements, it seems compulsory that the TM will always read the whole input in $f(n)$, so our superimposed finite automaton would get enough information to decide acceptance

(However, this argument has no connection to Papadimitriou's $2j$ bound)

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This leads to a generalised Gap Theorem:

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Example 13.9: There is a function f such that

$$\text{DTime}(f(n)) = \text{DTime} \left(\underbrace{2^{2^{\dots^2}}}_{f(n) \text{ times}} \right)$$

Moreover, the Gap Theorem can also be shown for space (and for other resources) in a similar fashion (space is a bit easier since the case of short words $|w| < j$ is easy to handle in very little space)

Discussion: Significance of the Gap Theorem

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¹Allender, Loui, Reagan: Complexity Theory. In Computing Handbook, 3rd ed., CRC Press, 2014)

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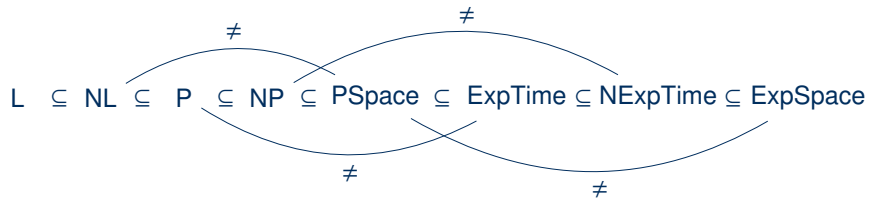
“Fortunately, the gap phenomenon cannot happen for time bounds t that anyone would ever be interested in”¹

Main insight: better stick to constructible functions

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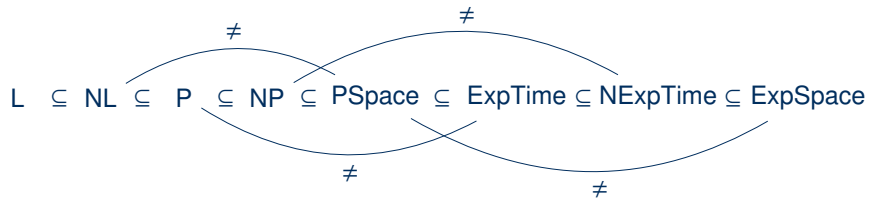
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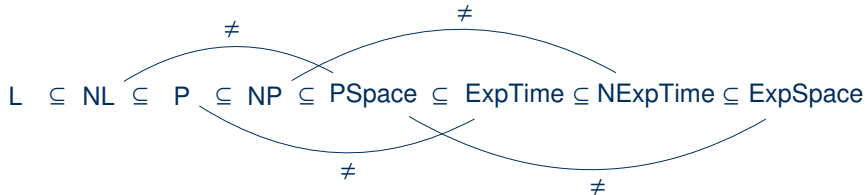
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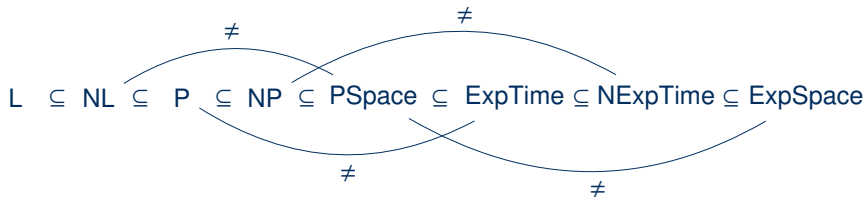


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With non-constructible functions as time/space bounds, arbitrary (constructible or not) boosts in resources do not lead to more power

Summary and Outlook

Hierarchy theorems tell us that more time/space leads to more power:



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With non-constructible functions as time/space bounds, arbitrary (constructible or not) boosts in resources do not lead to more power

What's next?

- Computing with oracles (reprise)
- The limits of diagonalisation, proved by diagonalisation
- P vs. NP again