Technische Universität Dresden

Prof. Dr. Sebastian Rudolph

Introduction to Formal Concept Analysis

Exercise Sheet 8, Winter Semester 2017/18

Exercise 1 (repetition)

Discuss with your neighbor the following concepts

- *closure system* and *closure operator*
- *frequent* concept intent
- *minimal generator*
- *implication* in a formal context $\mathbb{K} = (G, M, I)$
- *closed*, *complete* and *non-redundant* set of implications
- *stem base*

Further, describe the TITANIC algorithmus in three short sentences.

Exercise 2 (pseudo-closed sets)

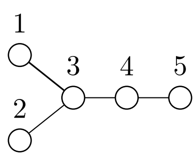
In the lecture the concept of *pseudo intents* was introduced. The following definition generalizes this concept in the context of closure systems:

Definition (pseudo-closed set). *Let $\mathcal{C}$ be a closure system on (the finite set) M. A subset $P \subseteq M$ is* pseudo-closed, *iff*

(i) P is not closed (i.e., $P \notin \mathcal{C}$), and

(ii) for every proper pseudo-closed subset $Q \subset P$, its closure $\varphi(Q)$ is contained in P (i.e., $Q \subset P \wedge Q$ is pseudo-closed $\implies \varphi(Q) \subseteq P$).

We are now regarding for the set of nodes $M := \{1, 2, \ldots, 5\}$ and the following tree T



the system $\mathcal{T} \subseteq \mathfrak{P}(M)$ of sets of nodes, which span a subtree of T, respectively (e.g., $\{1, 3, 4\} \in \mathcal{T}$ but $\{1, 2, 5\} \notin \mathcal{T}$).

a) Specify the set $\mathcal{T}$.

b) Verify that $\mathcal{T}$ is a closure system on M.

c) List six different pseudo-closed sets for $\mathcal{T}$.

Solution:

a)

$$\mathcal{T} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1,3\}, \{3,4\}, \{2,3\}, \{4,5\}, \{1,2,3\}, \{1,3,4\}, \{2,3,4\}, \{3,4,5\}, \{1,2,3,4\}, \{1,3,4,5\}, \{2,3,4,5\}, \{1,2,3,4,5\}\}$$

b) Using exhaustive enumeration: In the following table a cross indicates the intersection belongs to the set $\mathcal{T}$. The table is symmetric across the main diagonal.

$\cap$	{1}	{2}	{3}	{4}	{5}	{1,3}	{2,3}	{3,4}	{4,5}	{1,2,3}	{1,3,4}	{2,3,4}	{3,4,5}	{1,2,3,4}	{1,3,4,5}	{2,3,4,5}	{1,2,3,4,5}
{1}	×																
{2}	×	×															
{3}	×	×	×														
{4}	×	×	×	×													
{5}	×	×	×	×	×												
{1,3}	×	×	×	×	×	×											
{2,3}	×	×	×	×	×	×	×										
{3,4}	×	×	×	×	×	×	×	×									
{4,5}	×	×	×	×	×	×	×	×	×								
{1,2,3}	×	×	×	×	×	×	×	×	×	×							
{1,3,4}	×	×	×	×	×	×	×	×	×	×	×						
{2,3,4}	×	×	×	×	×	×	×	×	×	×	×	×					
{3,4,5}	×	×	×	×	×	×	×	×	×	×	×	×	×				
{1,2,3,4}	×	×	×	×	×	×	×	×	×	×	×	×	×	×			
{1,3,4,5}	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×		
{2,3,4,5}	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	
{1,2,3,4,5}	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×	×

c) pseudo-closed sets for $\mathcal{T}$.

- Step 1: P is not closed. i.e $P \neq P''$. i.e $\mathfrak{P}(M) - \mathcal{T}$ Hence potentially any of the following: {1,2}, {1,4}, {1,5}, {2,4}, {2,5}, {3,5}, {1,2,4}, {1,2,5}, {1,3,5}, {2,3,5}, {1,4,5}, {2,4,5}, {1,2,4,5}, {1,2,3,5}.
- Step 2: If $Q \subset P$ is a pseudo-closed proper subset of P, then $Q'' \subseteq P$. Note, in this case the closure operator : *is a sub-tree of*
  {1,2}, {1,4}, {1,5}, {2,4}, {2,5}, {3,5}

Exercise 3 (computing the stem base with Next Closure)

Determine the stem base for this context using the NEXT CLOSURE algorithm. Use the following table as help:

	Mobil (1)	Telefon (2)	Fax (3)	Fax m. N.-Adapter (4)
Sinus 44 (a)		×		
Nokia 6110 (b)	×	×		
T-Fax 301 (c)			×	×
T-Fax 360 PC (d)			×	

A	i	$A+i$	$\mathcal{L}(A+i)$	$A<_i\mathcal{L}(A+i)?$	$(\mathcal{L}(A+i))''$	$\mathcal{L}$	intents

Solution:

A	i	$A \bullet i$	$\mathcal{L}(A \bullet i)$	$A <_i \mathcal{L}(A \bullet i)?$	$(\mathcal{L}(A \bullet i))''$	$\mathcal{L}$	Intents
						$\emptyset$	$\emptyset$
$\emptyset$	4	$\{4\}$	$\{4\}$	$\times$	$\{3,4\}$	$\{4\} \to \{3\}$	
$\{4\}$	3	$\{3\}$	$\{3\}$	$\times$	$\{3\}$		$\{3\}$
$\{3\}$	4	$\{3,4\}$	$\{3,4\}$	$\times$	$\{3,4\}$		$\{3,4\}$
$\{3,4\}$	2	$\{2\}$	$\{2\}$	$\times$	$\{2\}$		$\{2\}$
$\{2\}$	4	$\{2,4\}$	$\{2,3,4\}$				
	3	$\{2,3\}$	$\{2,3\}$	$\times$	$\{1,2,3,4\}$	$\{2,3\} \to \{1,4\}$	
$\{2,3\}$	4	$\{2,3,4\}$	$\{1,2,3,4\}$				
	1	$\{1\}$	$\{1\}$	$\times$	$\{1,2\}$	$\{1\} \to \{2\}$	
$\{1\}$	4	$\{1,4\}$	$\{1,2,3,4\}$				
	3	$\{1,3\}$	$\{1,2,3,4\}$				
	2	$\{1,2\}$	$\{1,2\}$	$\times$			$\{1,2\}$
$\{1,2\}$	4	$\{1,2,4\}$	$\{1,2,3,4\}$				
	3	$\{1,2,3\}$	$\{1,2,3,4\}$				$\{1,2,3,4\}$