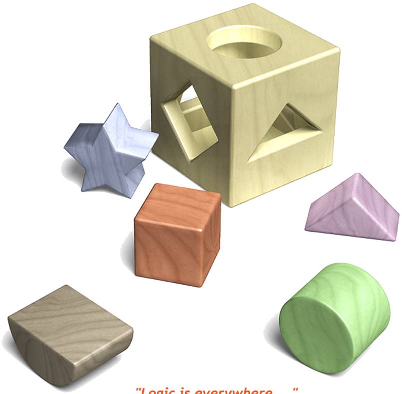
Weak Completion Semantics and Abduction

Steffen Hölldobler
International Center for Computational Logic
Technische Universität Dresden
Germany

- The Suppression Task
- Rules, Facts and Assumptions
- Weak Completion Semantics
- Abduction



"Logic is everywhere ..."

The Suppression Task

- Byrne: Suppressing Valid Inferences with Conditionals. Cognition 31, 61-83: 1989

C_1	**If she has an essay to write, then she will go to the library**
C_2	**if she has a textbook to read, then she will go to the library**
C_3	**If the library is open, then she will go to the library**
E	**She has an essay to write**
L	**She will go to the library**

	E	$\neg E$	L	$\neg L$
C_1	96%	46%	71%	92%
$C_1 \& C_2$	96%	4%	13%	96%
$C_1 \& C_3$	38%	63%	54%	33%
	(L)	$(\neg L)$	(E)	$(\neg E)$

The Suppression Task – Modus Ponens

- Stenning, van Lambalgen: Human Reasoning and Cognitive Science. MIT Press: 2008

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false

- **Weakly completed programs & least models**

$$e \leftrightarrow \top$$
$$\ell \leftrightarrow e \wedge \neg ab_1$$
$$ab_1 \leftrightarrow \bot$$

true	false	
e	ab_1	$\Phi \uparrow 1$
ℓ		$\Phi \uparrow 2$

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e, \ell\}, \{ab_1\} \rangle \models_{wcs} \ell$$

- Łukasiewicz: O logice trójwartościowej. Ruch Filozoficzny 5, 169-171: 1920
  H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics LNCS 5649, 464-478: 2009

The Suppression Task – Alternative Argument

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false
$\ell \leftarrow t \wedge \neg ab_2$	rule	definition of ℓ
$ab_2 \leftarrow \bot$	assumption	ab_2 is assumed to be false

- **Weakly completed programs & least models**

$$\begin{aligned} e &\leftrightarrow \top \\ \ell &\leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2) \\ ab_1 &\leftrightarrow \bot \\ ab_2 &\leftrightarrow \bot \end{aligned}$$

true	false	
e	ab_1	
	ab_2	$\Phi \uparrow 1$
ℓ		$\Phi \uparrow 2$

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e, \ell\}, \{ab_1, ab_2\} \rangle \models_{wcs} \ell$$

The Suppression Task – Additional Argument

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false
$\ell \leftarrow o \wedge \neg ab_3$	rule	definition of ℓ
$ab_3 \leftarrow \bot$	assumption	ab_3 is assumed to be false
$ab_1 \leftarrow \neg o$	rule	definition of ab_1
$ab_3 \leftarrow \neg e$	rule	definition of ab_3

- **Weakly completed programs & least models**

$$
\begin{aligned}
e &\leftrightarrow \top \\
\ell &\leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_3) \\
ab_1 &\leftrightarrow \bot \vee \neg o \\
ab_3 &\leftrightarrow \bot \vee \neg e
\end{aligned}
$$

true	*false*	
e		$\Phi \uparrow 1$
	ab_3	$\Phi \uparrow 2$

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e\}, \{ab_3\} \rangle \not\models_{wcs} \ell$$

The Suppression Task – Denial of the Antecedent

- **Programs**

$$\begin{array}{rcl} e & \leftarrow & \bot \\ \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array}$$

- **Weakly completed programs & least models**

$$\begin{array}{rcl} e & \leftrightarrow & \bot \\ \ell & \leftrightarrow & e \wedge \neg ab_1 \\ ab_1 & \leftrightarrow & \bot \end{array}$$

true	*false*
	e
	ab_1
	ℓ

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \emptyset, \{e, ab_1, \ell\} \rangle \models_{wcs} \neg \ell$$

The Suppression Task – Affirmation of the Consequent

- **Programs**

$$\begin{array}{rcl} \ell & \leftarrow & \top \\ \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array}$$

- **Weakly completed programs & least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & \top \vee (e \wedge \neg ab_1) \\ ab_1 & \leftrightarrow & \bot \end{array}$$

true	*false*
ℓ	ab_1

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{\ell\}, \{ab_1\} \rangle \not\models_{wcs} e$$

  - Byrne 1989 **most humans conclude** e**!**

Abduction

- **Programs & observations**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array}$$

ℓ

- **Abducibles**

$$e \leftarrow \top \qquad e \leftarrow \bot$$

- **Weakly completed programs plus explanations & least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & e \wedge \neg ab_1 \\ ab_1 & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \end{array}$$

true	*false*
e	ab_1
ℓ	

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e, \ell\}, \{ab_1\} \rangle \models_{wcs} e$$

- H., Philipp, Wernhard: An Abductive Model for Human Reasoning. In: Proc. Tenth Int. Symposium on Logical Formalizations of Commonsense Reasoning: 2011

Alternative Arguments and Affirmation of the Consequent

- **Programs & observations**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \\ \ell & \leftarrow & t \wedge \neg ab_2 \\ ab_2 & \leftarrow & \bot \end{array}$$

ℓ

- **Abducibles**

$e \leftarrow \top \qquad t \leftarrow \top \qquad e \leftarrow \bot \qquad t \leftarrow \bot$

- **Weakly completed programs plus explanations & least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2) \\ ab_1 & \leftrightarrow & \bot \\ ab_2 & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \quad \text{or} \quad t \leftrightarrow \top \end{array}$$

true	false
e	ab_1
	ab_2
ℓ	

true	false
t	ab_1
	ab_2
ℓ	

- **Computing skeptical consequences with respect to both models**
  - e does not follow

- Dietz, H., Ragni: A Computational Logic Approach to the Suppression Task Proc. COGSCI, 1500-1505: 2012

Usually Birds Fly

- **Programs & observations**

$$
\begin{aligned}
fly(X) &\leftarrow bird(X) \wedge \neg ab(X) \\
ab(X) &\leftarrow \bot \\
bird(tweety) &\leftarrow \top \\
bird(jerry) &\leftarrow \top
\end{aligned}
\qquad \neg fly(tweety)
$$

- **Abducibles**

$$\emptyset$$

- **Weakly completed programs plus explanations & least models**

$$
\begin{aligned}
fly(tweety) &\leftrightarrow bird(tweety) \wedge \neg ab(tweety) \\
fly(jerry) &\leftrightarrow bird(jerry) \wedge \neg ab(jerry) \\
ab(tweety) &\leftrightarrow \bot \\
ab(jerry) &\leftrightarrow \bot \\
bird(tweety) &\leftrightarrow \top \\
bird(jerry) &\leftrightarrow \top
\end{aligned}
$$

true	*false*
$bird(tweety)$	$ab(tweety)$
$bird(jerry)$	$ab(jerry)$
$fly(tweety)$	
$fly(jerry)$	

  - **We cannot explain the observation!**

Abducibles Revised

- **Programs & observations**

$$\begin{array}{rcl} fly(X) & \leftarrow & bird(X) \wedge \neg ab(X) \\ ab(X) & \leftarrow & \bot \\ bird(tweety) & \leftarrow & \top \\ bird(jerry) & \leftarrow & \top \end{array} \qquad \neg fly(tweety)$$

- **Abducibles**

$$ab(tweety) \leftarrow \top \qquad ab(jerry) \leftarrow \top$$

- **Weakly completed programs plus explanations & least models**

$$\begin{array}{rcl} fly(tweety) & \leftrightarrow & bird(tweety) \wedge \neg ab(tweety) \\ fly(jerry) & \leftrightarrow & bird(jerry) \wedge \neg ab(jerry) \\ ab(tweety) & \leftrightarrow & \bot \vee \top \\ ab(jerry) & \leftrightarrow & \bot \\ bird(tweety) & \leftrightarrow & \top \\ bird(jerry) & \leftrightarrow & \top \end{array}$$

true	false
$bird(tweety)$	$ab(jerry)$
$bird(jerry)$	
$ab(tweety)$	
$fly(jerry)$	$fly(tweety)$

- Dietz Saldanha, H., Pereira
  Contextual Reasoning: Usually Birds can Abductively Fly. TUD: 2017 (submitted)

Summary

- **Usually,**
  - $\mathcal{P}$ **denotes a program, i.e. a finite set of facts, rules and assumptions**
  - g$\mathcal{P}$ **denotes the ground instance of** $\mathcal{P}$
  - $wc\mathcal{P}$ **denotes the weak completion of** g$\mathcal{P}$
  - $\mathcal{M}_{\mathcal{P}}$ **denotes the least model of** $wc\mathcal{P}$
  - $\mathcal{A}_{\mathcal{P}}$ **denotes the set of abducibles of** g$\mathcal{P}$
  - $\mathcal{IC}$ **denotes a finite set of integrity constraints of the form** $\mathsf{U} \leftarrow L_1 \wedge \ldots \wedge L_n$
- **Weak Completion Semantics is the approach**
  - **to consider weakly completed programs**
  - **to compute their least models**
  - **and to reason with respect to these models**