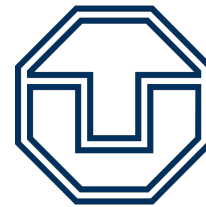


# Reasoning with Description Logics Ontologies and Knowledge Graphs

David Carral



**TECHNISCHE  
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DRESDEN**

Slides available at <https://iccl.inf.tu-dresden.de/web/Paristech-invited-talk>

Motivation

# Knowledge Graphs



# What is a Knowledge Graph?

A Knowledge Graph is a data repository that is:

- \* **Normalised:** Data is decomposed into small units (“edges”)
- \* **Connected:** Knowledge is represented by relationships between these units

Extending KGs with **OWL**  
**terminological axioms:**

- \* Data integration
- \* Information extraction
- \* Automatic population
- \* Debugging



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# Scalability

KGs contain **large amounts of assertional information**:



$50 \times 10^6$  statements

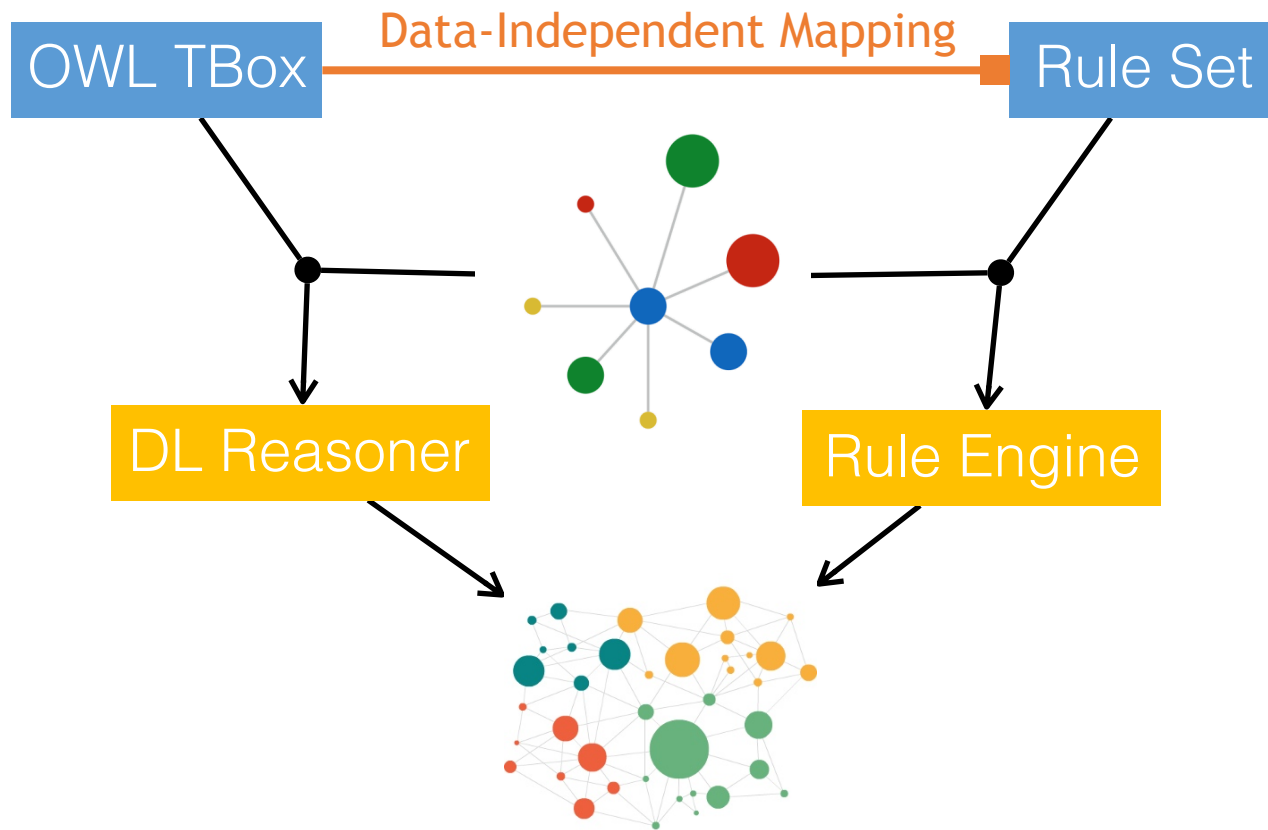


$70 \times 10^9$  facts,  $> 570 \times 10^6$  entities

Lack of tool support:

- \* **Traditional KR/DL View: Schema first** (class consistency, classification...)
- \* **Knowledge Graphs: Data first** (instance retrieval, CQ answering...)

# From OWL to Rules



In **theory**:

- \* Correctness
- \* Complexity

In **practice**:

- \* Implement transformations
- \* Evaluate performance
- \* Further develop and optimise rule engines

# From OWL to Rules

## Acyclicity Notions

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## Datalog Rewritings

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## The Combined Approach

- \* Pushing the Boundaries of Tractable Ontology Reasoning [ISWC 2014]
- \* The Combined Approach to Query Answering Beyond the OWL 2 Profiles [IJCAI 2015]
- \* The Combined Approach to Query Answering Horn-ALCHOIQ [KR 2018]

# The Combined Approach to Query Answering Horn-*ALCHOIQ*

David Carral, Irina Dragoste, and  
Markus Krötzsch [KR 2018]



# The DL Horn-*ALCHOIQ*: Examples

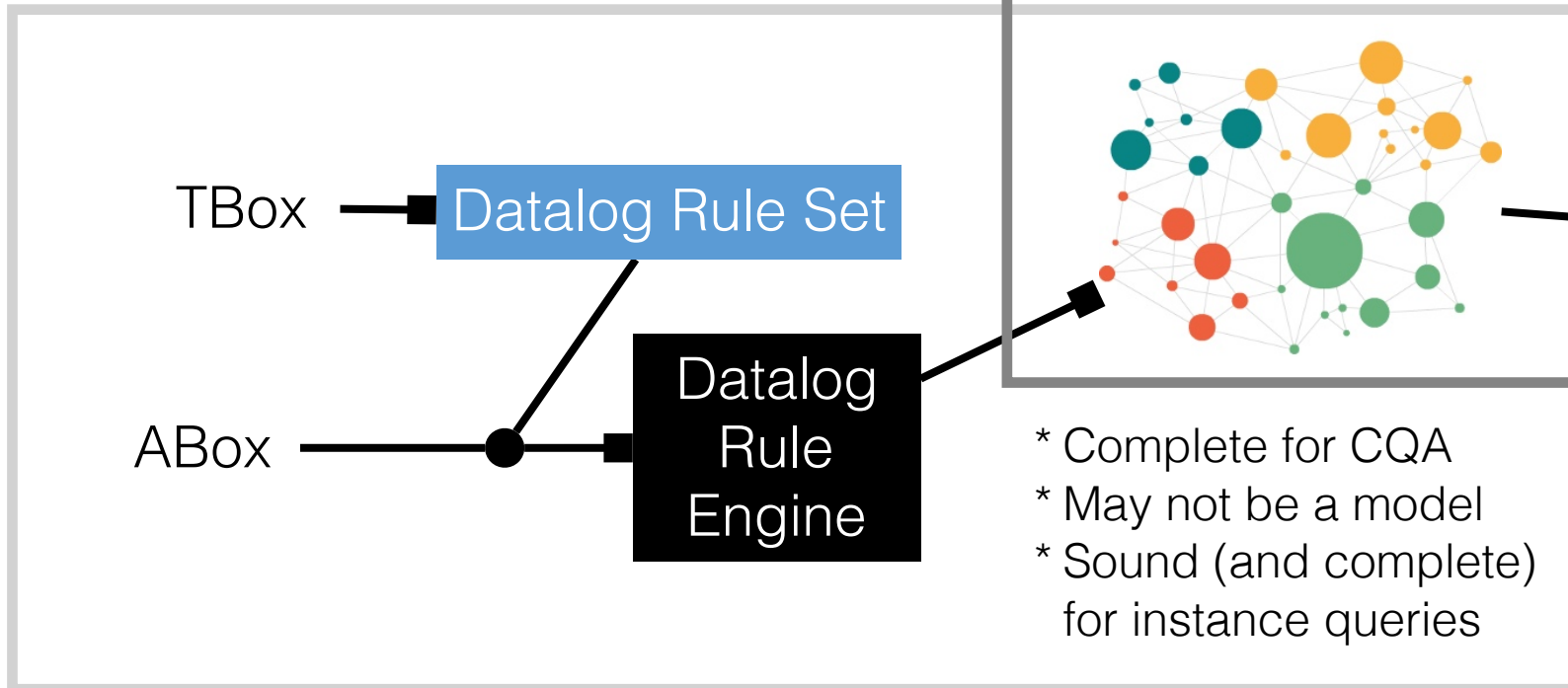
$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D$	$\rightarrow$	EnglishSpeaker $\sqcap$ FrenchSpeaker $\sqsubseteq$ Bilingual, Car $\sqsubseteq$ Vehicle
$\exists R. C \sqsubseteq D$	$\rightarrow$	$\exists$ Attends . Lecture $\sqsubseteq$ Student
$C \sqsubseteq \exists R. D$	$\rightarrow$	Bycycle $\sqsubseteq \exists$ HasPart . Wheel
$C \sqsubseteq \leq 1R. D$	$\rightarrow$	Course $\sqsubseteq \leq 1$ IsTaughtBy . Lecturer
$C \sqsubseteq \{a\}$	$\rightarrow$	Greek $\sqsubseteq \exists$ IsACitizenOf . {Greece}
$R \sqsubseteq S$	$\rightarrow$	HasFriend $\sqsubseteq$ Knows,
$R^- \sqsubseteq S$	$\rightarrow$	Supervises <sup>-</sup> $\sqsubseteq$ IsSupervisedBy
$R_1 \sqcap \dots \sqcap R_n \sqsubseteq \perp$	$\rightarrow$	Supervises $\sqcap$ IsSupervisedBy $\sqsubseteq \perp$
$C(a)$	$\rightarrow$	Student(joe)
$R(a, b)$	$\rightarrow$	HasSibling(joe, mike)

# The DL Horn-*ALCHOTIQ*: Semantics

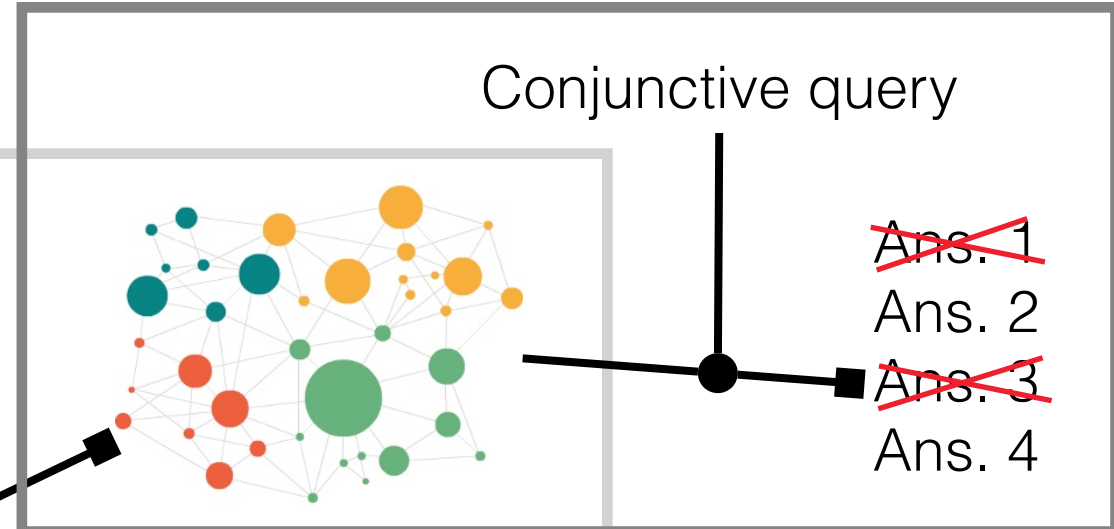
$$\begin{aligned} C_1 \sqcap \dots \sqcap C_n \sqsubseteq D &\mapsto C_1(x) \wedge \dots \wedge C_n(x) \rightarrow D(x) \\ \exists R. C \sqsubseteq D &\mapsto R(x, y) \wedge C(y) \rightarrow D(x) \\ C \sqsubseteq \exists R. D &\mapsto C(x) \rightarrow \exists y. R(x, y) \wedge D(y) \\ C \sqsubseteq \leq 1 R. D &\mapsto C(x) \wedge R(x, y) \wedge D(y) \wedge R(x, z) \wedge D(z) \rightarrow y \approx z \\ C \sqsubseteq \{a\} &\mapsto C(x) \rightarrow a \approx x \\ R \sqsubseteq S &\rightarrow R(x, y) \rightarrow S(x, y) \\ R^- \sqsubseteq S &\rightarrow R(y, x) \rightarrow S(x, y) \\ R_1 \sqcap \dots \sqcap R_n \sqsubseteq \perp &\rightarrow R_1(x, y) \wedge \dots \wedge R_n(x, y) \rightarrow \perp(x) \\ C(a) &\rightarrow A(a) \\ R(a, b) &\rightarrow R(a, b) \end{aligned}$$

# The Combined Approach

## 1. Materialisation Step



## 2. Filtration Step



# The Materialisation Step for $\mathcal{EL}$

$$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D \mapsto C_1(x) \wedge \dots \wedge C_n(x) \rightarrow D(x)$$

$$\exists R. C \sqsubseteq D \mapsto R(x, y) \wedge C(y) \rightarrow D(x)$$

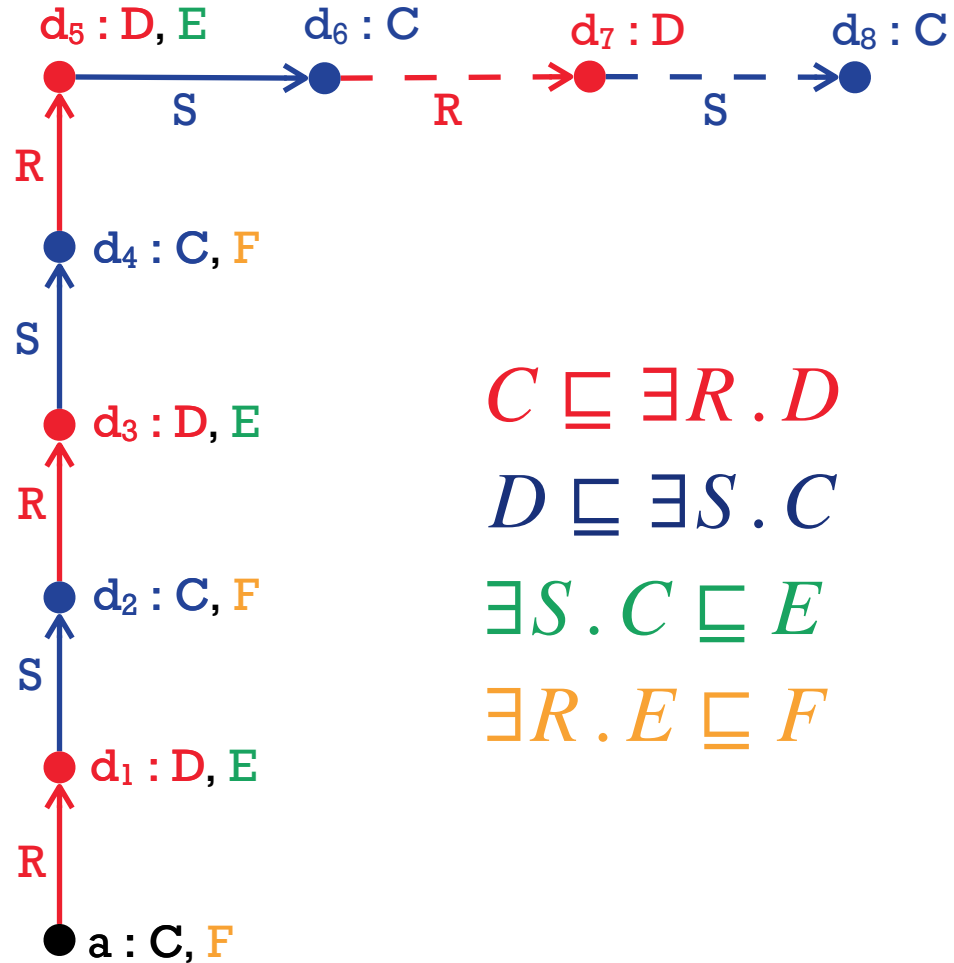
$$C \sqsubseteq \exists R. D \mapsto C(x) \rightarrow R(x, t_D) \wedge D(t_D)$$

$$R \sqsubseteq S \mapsto R(x, y) \rightarrow S(x, y)$$

$$C(a) \mapsto C(a)$$

$$R(a, b) \mapsto R(a, b)$$

# The Materialisation Step for $\mathcal{EL}$



$$C \sqsubseteq \exists R . D \quad C(a)$$

$$D \sqsubseteq \exists S . C$$

$$\exists S . C \sqsubseteq E$$

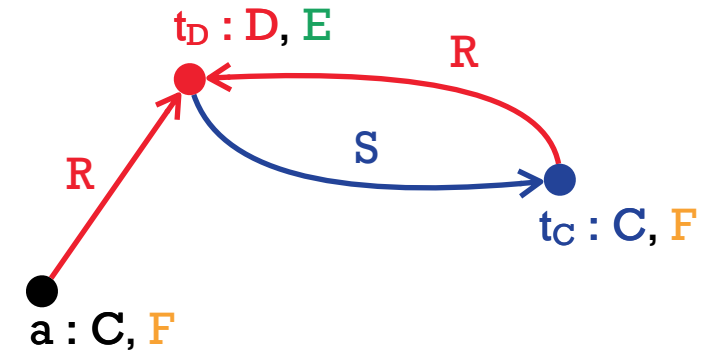
$$\exists R . E \sqsubseteq F$$

$$C(x) \rightarrow R(x, t_D) \wedge D(t_D)$$

$$D(x) \rightarrow S(x, t_C) \wedge C(t_C)$$

$$S(x, y) \wedge C(y) \rightarrow E(x)$$

$$R(x, y) \wedge E(y) \rightarrow F(x)$$



# The Materialisation Step for $\mathcal{ELI}$

$$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D \quad \mapsto \quad C_1(x) \wedge \dots \wedge C_n(x) \rightarrow D(x)$$

$$\exists R . C \sqsubseteq D \quad \mapsto \quad R(x, y) \wedge C(y) \rightarrow D(x),$$

$$C(x) \wedge R^-(x, t_{\mathbb{E}}) \rightarrow R^-(x, t_{\mathbb{E} \sqcap D}) \wedge \bigwedge_{E \in \mathbb{E} \sqcap D} E(t_{\mathbb{E} \sqcap D})$$

for every conjunction  $\mathbb{E}$  of concept names

$$C \sqsubseteq \exists R . D \quad \mapsto \quad C(x) \rightarrow R(x, t_D) \wedge D(t_D)$$

$$R \sqsubseteq S \quad \mapsto \quad R(x, y) \rightarrow S(x, y), R^-(x, y) \rightarrow S^-(x, y)$$

$$R^- \sqsubseteq S \quad \mapsto \quad R(x, y)^- \rightarrow S(x, y), R(x, y) \rightarrow S^-(x, y)$$

# The Materialisation Step for Horn- $\mathcal{ELI}$

$C \sqsubseteq \exists R.D$     $\exists R^-.E \sqsubseteq F$     $C(a)$

$D \sqsubseteq \exists S.C$     $\exists S^-.F \sqsubseteq E$     $E(a)$

$\exists R.F \sqsubseteq G$     $C(b)$

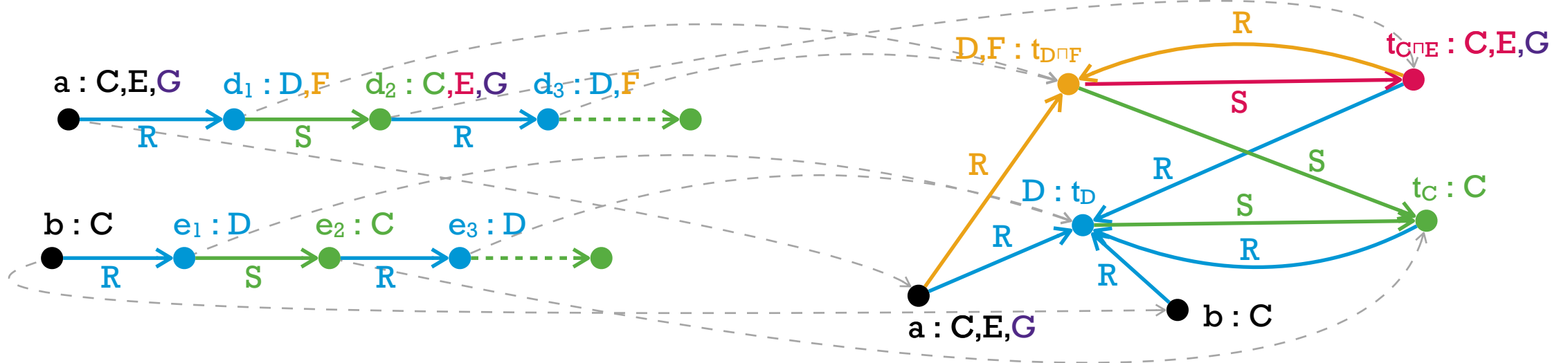
$C(x) \rightarrow R(x, t_D) \wedge D(t_D)$

$D(x) \rightarrow S(x, t_C) \wedge C(t_C)$

$E(x) \wedge R(x, t_D) \rightarrow R(x, t_{D \sqcap F}) \wedge D(t_{D \sqcap F}) \wedge F(t_{D \sqcap F})$

$F(x) \wedge S(x, t_C) \rightarrow S(x, t_{C \sqcap E}) \wedge C(t_{C \sqcap E}) \wedge E(t_{C \sqcap E})$

$R(x, y) \wedge F(y) \rightarrow G(x)$



$$q_1 = \exists y, w . C(a) \wedge R(a, y) \wedge S(y, w) \wedge G(w)$$

$$q_2 = \exists y, w . C(a) \wedge R(a, y) \wedge S(y, w) \wedge R(w, y)$$

# The Materialisation Step for Horn-*ALCHOIQ*

$$\mathcal{R}_{\text{Top}} = \{C(x) \rightarrow T(x) \mid C \in N_C\} \cup \{\mathbb{R}(x, y) \rightarrow T(x) \wedge T(y) \mid \mathbb{R} \in N_R^\square\}$$

$$\mathcal{R}_{\text{Role}} = \{\mathbb{R}(x, y) \wedge N(y) \rightarrow \mathbb{R}^-(x, y) \mid \mathbb{R} \in N_R^\square\} \cup \\ \{\mathbb{R}(x, y) \rightarrow R(x, y) \mid \mathbb{R} \in N_R^\square, R \in N_R\}$$

$$\mathcal{R}_{\text{Nm}} = \{N(a), T(a) \mid a \in N_I\}$$

$$\mathcal{R}_{\text{Eq}} = \{x \approx y \rightarrow y \approx x, x \approx y \wedge y \approx z \rightarrow x \approx z\} \cup \\ \{C(x) \wedge x \approx y \rightarrow C(y) \mid C \in N_C^+\} \cup \\ \{\mathbb{R}(x, y) \wedge x \approx z \rightarrow \mathbb{R}(z, y), \mathbb{R}(x, y) \wedge y \approx z \rightarrow \mathbb{R}(x, z) \mid \mathbb{R} \in N_R^\square\}$$



# The Materialisation Step for Horn-*ALCHQIQ*

$$C_1 \sqcap \dots \sqcap C_n \sqsubseteq D \quad \mapsto \quad C_1(x) \wedge \dots \wedge C_n(x) \rightarrow D(x)$$

$$\exists R. C \sqsubseteq D \quad \mapsto \quad R(x, y) \wedge C(y) \rightarrow D(x),$$

$$C(x) \wedge \mathbb{R}^-(x, t_{\mathbb{E}}) \rightarrow \mathbb{R}^-(x, t_{\mathbb{E} \sqcap D}) \wedge \bigwedge_{E \in \mathbb{E} \sqcap D} E(t_{\mathbb{E} \sqcap D})$$

for every  $\mathbb{R} \in N_R^\sqcap$  with  $R \in \mathbb{R}$  and  $\mathbb{E} \in N_C^\sqcap$

$$C \sqsubseteq \exists R. D \quad \mapsto \quad C(x) \rightarrow R(x, t_D) \wedge D(t_D)$$

$$R \sqsubseteq S \quad \mapsto \quad \mathbb{R}(x, y) \rightarrow (\mathbb{R} \sqcap S)(x, y), \mathbb{R}^-(x, y) \rightarrow (\mathbb{R}^- \sqcap S^-)(x, y)$$

for every  $\mathbb{R} \in N_R^\sqcap$  with  $R \in \mathbb{R}$

$$R^- \sqsubseteq S \quad \mapsto \quad \mathbb{R}^-(x, y) \rightarrow (\mathbb{R} \sqcap S)(x, y), \mathbb{R}(x, y) \rightarrow (\mathbb{R} \sqcap S^-)(x, y)$$

for every  $\mathbb{R} \in N_R^\sqcap$  with  $R \in \mathbb{R}$

$$C \sqsubseteq \{a\} \quad \mapsto \quad C(x) \rightarrow x \approx a$$

# The Materialisation Step for Horn-*ALCHOIQ*

$$C \sqsubseteq \leq 1R.D \mapsto$$

For all  $\mathbb{R}, \mathbb{S} \in N_R^\sqcap$  with  $R \in \mathbb{R}$  and  $R \in \mathbb{S}$ , and all  $\mathbb{E}, \mathbb{F} \in N_C^\sqcap$

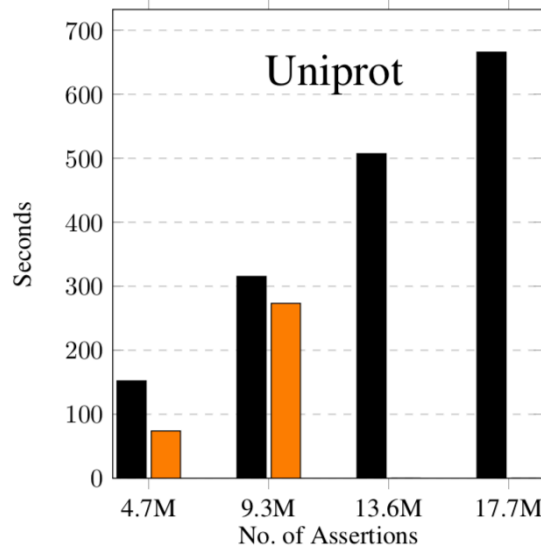
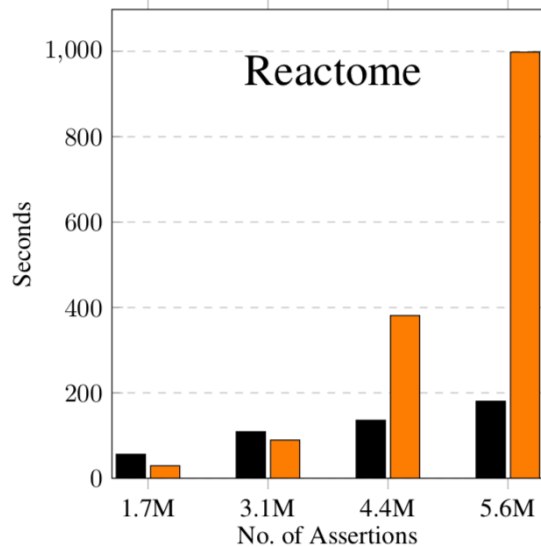
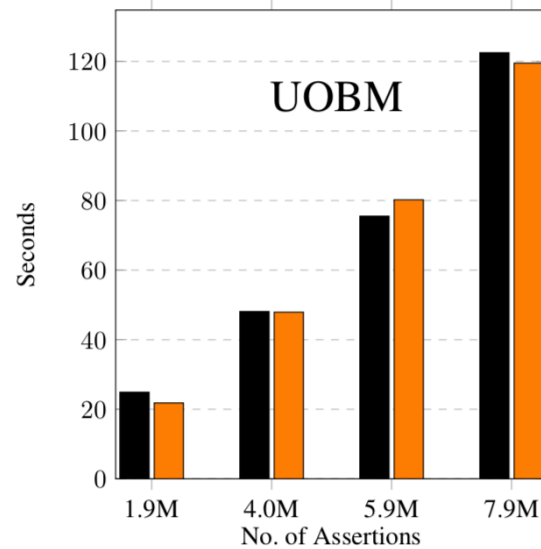
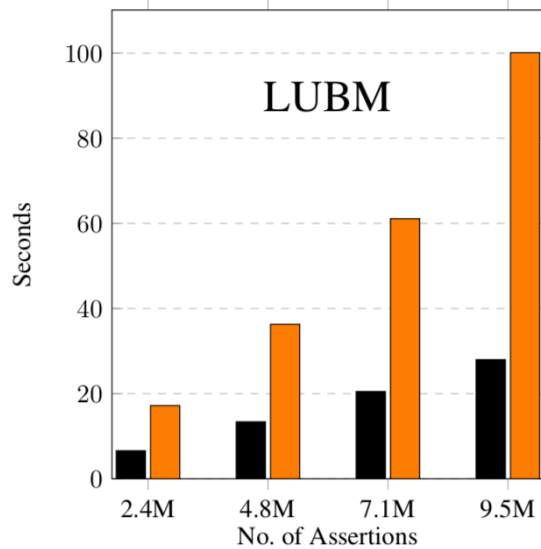
$$D(y) \wedge R^-(y, x) \wedge C(x) \wedge R(x, z) \wedge D(z) \wedge N(z) \rightarrow y \approx z$$

$$C(x) \wedge \mathbb{R}(x, t_{\mathbb{E}}) \wedge D(t_{\mathbb{E}}) \wedge \mathbb{S}(x, t_{\mathbb{F}}) \wedge F(t_{\mathbb{F}}) \rightarrow (\mathbb{R} \sqcap \mathbb{S})(x, t_{\mathbb{E} \sqcap \mathbb{F}}) \wedge \bigwedge_{X \in \mathbb{E} \sqcap \mathbb{F}} X(t_{\mathbb{E} \sqcap \mathbb{F}})$$

$$D(y) \wedge \mathbb{R}^-(y, x) \wedge C(x) \wedge \mathbb{S}(x, t_{\mathbb{E}}) \wedge D(t_{\mathbb{E}}) \rightarrow \bigwedge_{X \in \mathbb{E}} X(y) \wedge (\mathbb{R}^- \sqcap \mathbb{S}^-)(y, x)$$

$$D(y) \wedge R^-(y, x) \wedge C(x) \wedge N(x) \rightarrow N(y)$$

# Materialisation Step: Implementation and Evaluation

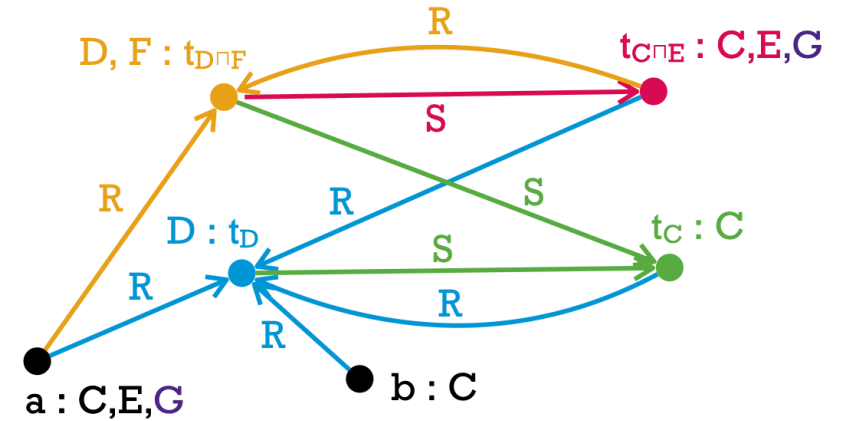


- Materialisation Step
- Konclude

- \* Use RDFox as a Datalog engine and add rules on demand
- \* Rule count per ontology:  
108+6 (LUBM), 254+19 (UOBM),  
481+14 (Reactome), 317+59 (Uniprot)
- \* RDFox uses renaming to deal with equality.

# Contributions

- \* We expand the combined approach to an expressive and non-tractable fragment such as Horn-*ALCHOIQ*
- \* Our method subsumes all previously existing combined approaches and is worst-case optimal.
- \* We produce the first implementation that solves CQ entailment over Horn-*ALCHOIQ* ontologies.



# Future work

- \* Extend our procedure in order to solve conjunctive regular path queries.
- \* Applying the above, we can produce an implementation for Horn-*SROIQ*.

# From OWL to Rules

## Acyclicity Notions

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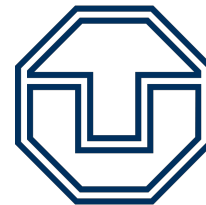
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$$C(x) \wedge \mathbb{R}(x, t_{\mathbb{E}}) \wedge D(t_{\mathbb{E}}) \wedge \mathbb{S}(x, t_{\mathbb{F}}) \wedge F(t_{\mathbb{F}}) \rightarrow (\mathbb{R} \sqcap \mathbb{S})(x, t_{\mathbb{E} \sqcap \mathbb{F}}) \wedge \bigwedge_{X \in \mathbb{E} \sqcap \mathbb{F}} X(t_{\mathbb{E} \sqcap \mathbb{F}})$$

$$C(x) \wedge (R \sqcap S)(x, t_{B \sqcap D}) \wedge D(t_{B \sqcap D})$$

$$\wedge R(x, t_A) \wedge D(t_A) \rightarrow (R \sqcap S)(x, t_{A \sqcap B \sqcap D}) \wedge A(t_{A \sqcap B \sqcap D}) \wedge B(t_{A \sqcap B \sqcap D}) \wedge D(t_{A \sqcap B \sqcap D})$$

