Problem 1.1

In the lectures the following example from Description Logics was presented:

\[
\begin{align*}
\mathcal{K}_T : & \quad \text{woman} \sqsubseteq \text{person}, \\
& \quad \text{man} \sqsubseteq \text{person}, \\
& \quad \text{mother} = \text{woman} \sqcap \exists \text{child} : \text{person}, \\
& \quad \text{father} = \text{man} \sqcap \exists \text{child} : \text{person}, \\
& \quad \text{parent} = \text{mother} \sqcup \text{father}, \\
& \quad \text{grandparent} = \text{parent} \sqcap \exists \text{child} : \text{parent}, \\
& \quad \text{father\_without\_son} = \text{father} \sqcap \forall \text{child} : \neg \text{man} \\
\end{align*}
\]

\[
\begin{align*}
\mathcal{K}_A : & \quad \text{parent(} \text{carl} \text{)}, \text{parent(} \text{conny} \text{)}, \\
& \quad \text{child(} \text{conny}, \text{joe} \text{)}, \text{child(} \text{conny}, \text{carl} \text{)}, \\
& \quad \text{man(} \text{joe} \text{)}, \text{man(} \text{carl} \text{)}, \text{woman(} \text{conny} \text{)}. \\
\end{align*}
\]

Are the following consequences valid? **Justify** your answers.

1. \(\mathcal{K}_T \cup \mathcal{K}_A \models \text{grandparent}\!(\text{conny})\)
2. \(\mathcal{K}_T \cup \mathcal{K}_A \models \text{father}\!(\text{carl})\)
3. \(\mathcal{K}_T \cup \mathcal{K}_A \models \text{father\_without\_son}\!(\text{carl})\)

Problem 1.2

Prove that \(F \sqsubseteq G \iff F \sqcap \neg G = \bot\).

Problem 1.3

Show that \(\text{grandparent} \sqsubseteq_{\mathcal{K}_T} \text{parent}\) by reducing subsumption into concept satisfiability, where \(\mathcal{K}_T\) is the T-Box from Problem 1.1.

Problem 1.4

Is the concept \((\text{father} \sqcap \text{mother})\) satisfiable w.r.t. \(\mathcal{K}_T\) from Problem 1.1?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?
2. Is there a single generalized concept axiom that prevents that a person is female and male?

Problem 1.6

Give an equivalent concept of \((\text{woman} \sqcap \exists \text{child}.\text{person})\) without using the constructors \(\sqcap\) and \(\exists r. C\)

Problem 1.7

Prove the following:

If \((\forall r. C)(a) \in A\), and \(r(a, b) \in A\), then \(A \models C(b)\).