EXERCISE 1

Science of Computational Logic

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European Master's Program in Computational Logic — winter semester 2017/2018

Problem 1.1

In the lectures the following example from Description Logics was presented:

$$
\begin{array}{lrcl}
\mathcal{K}_T : & \text{woman} & \sqsubseteq & \text{person}, \\
& \text{man} & \sqsubseteq & \text{person}, \\
& \text{mother} & = & \text{woman} \sqcap \exists \text{child} : \text{person}, \\
& \text{father} & = & \text{man} \sqcap \exists \text{child} : \text{person}, \\
& \text{parent} & = & \text{mother} \sqcup \text{father}, \\
& \text{grandparent} & = & \text{parent} \sqcap \exists \text{child} : \text{parent}, \\
& \text{father\_without\_son} & = & \text{father} \sqcap \forall \text{child} : \neg \text{man}
\end{array}
$$

$$
\begin{array}{ll}
\mathcal{K}_A : & \text{parent(carl)},\ \text{parent(conny)}, \\
& \text{child(conny, joe)},\ \text{child(conny, carl)}, \\
& \text{man(joe)},\ \text{man(carl)},\ \text{woman(conny)}.
\end{array}
$$

Are the following consequences valid? **Justify** your answers.

1. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{grandparent(conny)}$
2. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{father(carl)}$
3. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{father\_without\_son(carl)}$

Problem 1.2

Prove that $F \sqsubseteq G \;\equiv\; F \sqcap \neg G = \bot$

Problem 1.3

Show that grandparent $\sqsubseteq_{\mathcal{K}_T}$ parent by reducing subsumption into concept satisfiability, where $\mathcal{K}_T$ is the T-Box from Problem 1.1.

Problem 1.4

Is the concept (father $\sqcap$ mother) satisfiable w.r.t. $\mathcal{K}_T$ from Problem 1.1?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?

2. Is there a single generalized concept axiom that prevents that a person is female and male?

Problem 1.6

Give an equivalent concept of $(\text{woman} \sqcap \exists\text{child.person})$ without using the constructors $\sqcap$ and $\exists r.C$

Problem 1.7

Prove the following:

If $(\forall r.C)(a) \in \mathcal{A}$, and $r(a,b) \in \mathcal{A}$, then $\mathcal{A} \models C(b)$.