EXERCISE 5

Science of Computational Logic

Steffen Hölldobler, Tobias Philipp

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Problem 5.1

Consider the following theory:

$$\mathcal{E} = \{0 + X \approx X, s(X) + Y \approx s(X + Y)\}$$

1. Show that $\mathcal{E}_\approx \cup \mathcal{E} \not\models (\forall)(X + 0 \approx X)$
2. Specify a formula F such that $\mathcal{E}_\approx \cup \mathcal{E} \cup F \models (\forall)(X + 0 \approx X)$
3. Show that every minimal Herbrand model of $\mathcal{E}_\approx \cup \mathcal{E}$ is a model of the formula $(\forall)(X + 0 \approx X)$.
4. Show that every minimal Herbrand model of $\mathcal{E}_\approx \cup \mathcal{E}$ is a model of the formula $(\forall)(X + Y \approx Y + X)$.
5. Specify a formula $P(X)$ such that

$$\mathcal{E}_\approx \cup \mathcal{E} \cup \{(P(0) \wedge (\forall X)(P(X) \rightarrow P(s(X))) \rightarrow (\forall X)P(X)\} \models (\forall)(X + Y \approx Y + X)$$

Problem 5.2

The previous exercise defined the additon on natural numbers.

Specify a Horn-theory that defines the addition on integers.