Exercise 2: First-Order Queries

Database Theory

2020-04-20

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Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	**Address**	**Phone**
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
...	...	...

Program

Cinema	**Title**	**Time**
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
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Solution.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
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Program

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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of "The Imitation Game"?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
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Dogma	Smith	Smith

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Program

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Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of "The Imitation Game"?

$$\exists y_A. \text{Films}(\texttt{"The Imitation Game"}, x_D, y_A)[x_D]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

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Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of “The Imitation Game”?

$$\exists y_A.\, \text{Films}(\texttt{"The Imitation Game"}, x_D, y_A)[x_D]$$

2. Which cinemas feature “The Imitation Game”?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
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Program

Cinema	**Title**	**Time**
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of “The Imitation Game”?

$$\exists y_A.\ \mathrm{Films}(\texttt{"The Imitation Game"}, x_D, y_A)[x_D]$$

2. Which cinemas feature “The Imitation Game”?

$$\exists y_T.\ \mathrm{Program}(x_C, \texttt{"The Imitation Game"}, y_T)[x_C]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

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Program

Cinema	Title	Time
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Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of “Schauburg”?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of “Schauburg”?

$$\text{Venues}(\texttt{"Schauburg"}, x_A, x_P)[x_A, x_P]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

Venues

Cinema	**Address**	**Phone**
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Program

Cinema	**Title**	**Time**
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of "Schauburg"?

$$\text{Venues}(\texttt{"Schauburg"}, x_A, x_P)[x_A, x_P]$$

4. *Boolean query:* Is a film directed by "Smith" playing in Dresden?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

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Schauburg	The Imitation Game	19:30
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CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of "Schauburg"?

$$\text{Venues}(\texttt{"Schauburg"}, x_A, x_P)[x_A, x_P]$$

4. *Boolean query:* Is a film directed by "Smith" playing in Dresden?

$$\exists y_T, y_A, y_C, z_T.\ \text{Films}(y_T, \texttt{"Smith"}, y_A) \land \text{Program}(y_C, y_T, z_T)$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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Dogma	Smith	Damon
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Dogma	Smith	Smith

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Schauburg	The Imitation Game	19:30
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CinemaxX	The Imitation Game	19:30

Solution.

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\exists y_T, z_T.\ \mathrm{Films}(y_T, x_D, x_A) \wedge \mathrm{Films}(z_T, x_A, x_D)[x_D, x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\exists y_T, z_T.\, \text{Films}(y_T, x_D, x_A) \land \text{Films}(z_T, x_A, x_D)[x_D, x_A]$$

6. List the names of directors who have acted in a film they directed.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Program

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Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\exists y_T, z_T.\ \mathrm{Films}(y_T, x_D, x_A) \wedge \mathrm{Films}(z_T, x_A, x_D)[x_D, x_A]$$

6. List the names of directors who have acted in a film they directed.

$$\exists y_T.\ \mathrm{Films}(y_T, x_D, x_D)[x_D]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
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Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
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CinemaxX	The Imitation Game	19:30

Solution.

7. Always return $\{\text{Title} \mapsto \texttt{"Apocalypse Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

7. Always return $\{\text{Title} \mapsto \texttt{"Apocalypse Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

$$\big\{\text{DirectedBy}(\texttt{"Apocalypse Now"}, \texttt{"Coppola"})\big\}$$

Note: FO queries always use the unnamed perspective.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

7. Always return $\{\text{Title} \mapsto \texttt{"Apocalypse Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

$$\left\{\text{DirectedBy}(\texttt{"Apocalypse Now"}, \texttt{"Coppola"})\right\}$$

   Note: FO queries always use the unnamed perspective.

8. Find the actors cast in at least one film by "Smith".

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

7. Always return $\{\text{Title} \mapsto \texttt{"Apocalypse Now"}, \text{Director} \mapsto \texttt{"Coppola"}\}$ as the answer.

$$\left\{\text{DirectedBy}(\texttt{"Apocalypse Now"}, \texttt{"Coppola"})\right\}$$

   Note: FO queries always use the unnamed perspective.

8. Find the actors cast in at least one film by “Smith”.

$$\exists y_T.\ \text{Films}(y_T, \texttt{"Smith"}, x_A)[x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

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Program

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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

$$\exists y_T, y_D. \big(\mathrm{Films}(y_T, y_D, x_A) \land \forall z_T, z_A. (\mathrm{Films}(z_T, \texttt{"Smith"}, z_A) \to \mathrm{Films}(z_T, \texttt{"Smith"}, x_A))\big)[x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

$$\exists y_T, y_D. \Big(\text{Films}(y_T, y_D, x_A) \wedge \forall z_T, z_A. \left(\text{Films}(z_T, \texttt{"Smith"}, z_A) \rightarrow \text{Films}(z_T, \texttt{"Smith"}, x_A)\right)\Big)[x_A]$$

10. Find the actors cast only in films by "Smith."

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
...	...	...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

$$\exists y_T, y_D. \big(\text{Films}(y_T, y_D, x_A) \land \forall z_T, z_A. (\text{Films}(z_T, \texttt{"Smith"}, z_A) \to \text{Films}(z_T, \texttt{"Smith"}, x_A))\big)[x_A]$$

10. Find the actors cast only in films by "Smith."

$$\exists y_T, y_D. \big(\text{Films}(y_T, y_D, x_A) \land \forall z_T. \exists z_D. (\text{Films}(z_T, z_D, x_A) \to z_D \approx \texttt{"Smith"})\big)[x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Program

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Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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...	...	...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

$$\exists y_T, y_D.\ \mathrm{Films}(y_T, y_D, x_A) \wedge \mathrm{Films}(y_T, y_D, x_{A'}) \wedge x_A \not\approx x_{A'}\,[x_A, x_{A'}]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Program

Cinema	**Title**	**Time**
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

$$\exists y_T, y_D.\, \text{Films}(y_T, y_D, x_A) \wedge \text{Films}(y_T, y_D, x_{A'}) \wedge x_A \not\approx x_{A'}\, [x_A, x_{A'}]$$

12. Find all pairs of actors cast in exactly the same films.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	**Director**	**Actor**
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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...	...	...

Program

Cinema	**Title**	**Time**
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

$$\exists y_T, y_D. \text{Films}(y_T, y_D, x_A) \wedge \text{Films}(y_T, y_D, x_{A'}) \wedge x_A \not\approx x_{A'} [x_A, x_{A'}]$$

12. Find all pairs of actors cast in exactly the same films.

$$\exists y_T, y_D. \Big(\text{Films}(y_T, y_D, x_A) \wedge \exists z_T, z_D. \text{Films}(z_T, z_D, x_{A'}) \wedge \forall w, v. \big(\text{Films}(w, v, x_A) \leftrightarrow \text{Films}(w, v, x_{A'})\big)\Big)[x_A, x_{A'}]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

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...	...	...

Program

Cinema	Title	Time
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

13. Find the directors such that every actor is cast in one of their films.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...	...	...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...	...	...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

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Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

13. Find the directors such that every actor is cast in one of their films.

$$\exists y_T, y_A. \big(\mathrm{Films}(y_T, x_D, y_A) \wedge \forall z_T, z_D, z_A. \big(\mathrm{Films}(z_T, z_D, z_A) \rightarrow \exists w_T.\mathrm{Films}(w_T, x_D, z_A)\big)\big)[x_D]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a $\text{DI}_{\text{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\mathrm{named}}$ query as a $\mathrm{DI}_{\mathrm{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\text{named}}$ query as a $\mathrm{DI}_{\text{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;

  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$ with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\mathrm{named}}$ query as a $\mathrm{DI}_{\mathrm{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;

  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$ with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\ R(x_A, y_B)[x_A]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\mathrm{named}}$ query as a $\mathrm{DI}_{\mathrm{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;

  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$ with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\ R(x_A, y_B)[x_A]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A.\ R(y_A, x_B)[x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\mathrm{named}}$ query as a $\mathrm{DI}_{\mathrm{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;
  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$
  with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\ R(x_A, y_B)[x_A]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A.\ R(y_A, x_B)[x_B]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\mathrm{named}}$ query as a $\mathrm{DI}_{\mathrm{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- if $q = \delta_{B_1,\dots,B_n \to A_1,\dots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}.\, (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;

  Assumption: $A_1, \dots, A_n$ in $\delta_{B_1,\dots,B_n \to A_1,\dots,A_n}$ are written in attribute order; $B_1, \dots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\dots,A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}.\, \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\, R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A.\, R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\text{named}}$ query as a $\mathrm{DI}_{\text{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;
  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$
  with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\ R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{(\delta_{B,A \to A,B}(R))}[x_A, x_B] = \exists y_B, y_A.\ (x_A \approx y_B) \wedge (x_B \approx y_A) \wedge R(y_A, y_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A.\ R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\mathrm{named}}$ query as a $\mathrm{DI}_{\mathrm{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A \to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;

  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$ with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\ R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{(\delta_{B,A \to A,B}(R))}[x_A, x_B] = \exists y_B, y_A.\ (x_A \approx y_B) \wedge (x_B \approx y_A) \wedge R(y_A, y_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A.\ R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

$$\varphi_{R \bowtie (\delta_{B,A \to A,B}(R))}[x_A, x_B] = \varphi_R \wedge \varphi_{(\delta_{B,A \to A,B}(R))}[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following $\mathrm{RA}_{\text{named}}$ query as a $\mathrm{DI}_{\text{unnamed}}$ query:

$$q[A, B] = \big(\pi_A(R) \bowtie \pi_B(R)\big) - \big(R \bowtie (\delta_{B,A\to A,B}(R))\big)$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \ldots, a_n]$, let $\varphi_q[x_{a_1}, \ldots, x_{a_n}]$ be the DI query defined as follows:

- If $q = R$ with signature $R[A_1, \ldots, A_n]$, then $\varphi_q = R(x_{A_1}, \ldots, x_{A_n})[x_{A_1}, \ldots, x_{A_n}]$;
- if $q = \delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n} q'$, then $\varphi_q = \exists y_{B_1}, \ldots, y_{B_n}.\ (x_{A_1} \approx y_{B_1}) \wedge \ldots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \ldots, y_{A_n}]$;
  Assumption: $A_1, \ldots, A_n$ in $\delta_{B_1,\ldots,B_n \to A_1,\ldots,A_n}$ are written in attribute order; $B_1, \ldots, B_n$ may be in arbitrary order.
- if $q = \pi_{A_1,\ldots,A_n}(q')$ for a subquery $q'[B_1, \ldots, B_m]$
  with $\{B_1, \ldots, B_m\} = \{A_1, \ldots, A_n\} \cup \{C_1, \ldots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \ldots, x_{C_k}.\ \varphi_{q'}$;
- if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg\varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B.\ R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

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$$\varphi_{q[A,B]}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)} \wedge \neg\Big(\varphi_R \wedge \varphi_{(\delta_{B,A\to A,B}(R))}\Big)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A.\ R(y_A, x_B)[x_B]$$

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Exercise. It was stated in the lecture (Lecture 2, slide 17) that query mappings under named perspective can be translated into query mappings under unnamed perspective. Specify this translation.

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- Let $\mathbb{D}_n$ and $\mathbb{D}_u$ be the sets of all database instances over a named and an unnamed perspective, respectively, and let $\mathbb{T}_n$ and $\mathbb{T}_u$ be the sets of all database tables over a named and an unnamed perspective.

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- Consider a query mapping $M[q] : \mathbb{D}_n \to \mathbb{T}_n$.
- Define $\nu : \mathbb{T}_n \to \mathbb{T}_u$ as the function taking named database tables $R[A_1, \ldots, A_n]$ to unnamed database tables $R^{\mathcal{J}}$, such that attribute A_i is mapped to column i:

$$\nu(R^{\mathcal{I}}) = \left\{ \langle r(A_1), \ldots, r(A_n) \rangle \,\middle|\, (r : \{ A_1, \ldots, A_n \} \to \mathbf{dom}) \in R^{\mathcal{I}} \right\}$$

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- Conversely, let $\mu : \mathbb{D}_u \to \mathbb{D}_n$ be the function taking unnamed database instances $\mathcal{J}$ to named database instances $\mathcal{I}$, by mapping each table $R^{\mathcal{J}}$ to a named table taking attribute A_i from column i:

$$\mu(\mathcal{J}) = \left\{ \left\{ \{ A_1 \mapsto a_1, \ldots, A_n \mapsto a_n \} \mid \langle a_1, \ldots, a_n \rangle \in R^{\mathcal{J}} \right\} \,\middle|\, R \in \mathcal{J} \right\}$$

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- Then $\nu \circ M[q] \circ \mu : \mathbb{D}_u \to \mathbb{T}_u$ is the required translation of $M[q] : \mathbb{D}_n \to \mathbb{T}_n$.

$$\begin{tikzcd}
\mathbb{D}_n \arrow[r, "{M[q]}"] & \mathbb{T}_n \arrow[d, "\nu"] \\
\mathbb{D}_u \arrow[u, "\mu"] \arrow[r, "{\nu \circ M[q] \circ \mu}"] & \mathbb{T}_u
\end{tikzcd}$$

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Exercise. Complete the proof that $\mathsf{RA}_{\text{named}} \sqsubseteq \mathsf{DI}_{\text{unnamed}}$ by showing that the results of the transformation are (a) *domain independent* and (b) equivalent to the input query. In each case, show that the claimed property holds true for each case of the recursive construction under the assumption (induction hypothesis) that it has been established for all subqueries. Use the mappings from the previous exercise to compare named and unnamed results.

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- If $q = \delta_{b_1, \ldots, b_n \to a_1, \ldots, a_n} q'$, then $\varphi_q = \exists y_{b_1}, \ldots, y_{b_n}. (x_{a_1} \approx y_{b_1}) \wedge \ldots \wedge (x_{a_n} \approx y_{b_n}) \wedge \varphi_{q'}[y_{B_1}, \ldots, y_{B_n}]$.

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Exercise. Consider a binary predicate R and the $\mathrm{AD}_{\mathrm{unnamed}}$ query

$$\varphi[x, y] = \neg(R(x, y) \wedge R(y, x)).$$

Use the construction from the lecture to express it as an $\mathrm{RA}_{\mathrm{named}}$ query.

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\end{aligned}$$

Exercise 6

Exercise. Complete the constructions for the proof of AD $\sqsubseteq$ RA given in the lecture.

1. Define the relational algebra expression $E_{a,\mathbf{adom}}$, such that $E_{a,\mathbf{adom}}(\mathcal{I}) = \{\{a \mapsto c\} \mid c \in \mathbf{adom}(\mathcal{I}, q)\}$ (assume that the query and the database schema are known).
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$$E_{\varphi_1 \vee \varphi_2} = E_{a_{x_1},\textbf{adom}} \bowtie \cdots \bowtie E_{a_{x_n},\textbf{adom}} - E_{\neg\varphi_1 \wedge \neg\varphi_2}$$

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$$\begin{aligned} E_{\varphi_1 \vee \varphi_2} &= E_{a_{x_1},\,\mathbf{adom}} \bowtie \cdots \bowtie E_{a_{x_n},\,\mathbf{adom}} - E_{\neg\varphi_1 \wedge \neg\varphi_2} \\ &= E_{a_{x_1},\,\mathbf{adom}} \bowtie \cdots \bowtie E_{a_{x_n},\,\mathbf{adom}} - (E_{\neg\varphi_1} \bowtie E_{\neg\varphi_2}) \end{aligned}$$

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&= E_{a_{x_1},\mathbf{adom}} \bowtie \cdots \bowtie E_{a_{x_n},\mathbf{adom}} - ((E_{a_{y_1},\mathbf{adom}} \bowtie \cdots \bowtie E_{a_{y_\ell},\mathbf{adom}} - E_{\varphi_1}) \bowtie (E_{a_{z_1},\mathbf{adom}} \bowtie \cdots \bowtie E_{a_{z_k},\mathbf{adom}} - E_{\varphi_2}))
\end{aligned}$$

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{\mathsf{SID}}, y_{\mathsf{Stop}}, y_{\mathsf{To}}. \big(\mathsf{Stops}(y_{\mathsf{SID}}, y_{\mathsf{Stop}}, \texttt{"true"}) \wedge \mathsf{Connect}(y_{\mathsf{SID}}, y_{\mathsf{To}}, x_{\mathsf{Line}})\big)[x_{\mathsf{Line}}]$
2. $\varphi_2 = \neg\mathsf{Lines}(x, \texttt{"bus"})[x]$
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Which of these queries is a safe-range query? Which of the queries is domain independent?

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Definition (Lecture 2, Slide 26)

The set $\mathrm{rr}(\varphi)$ of range-restricted variables of φ in Safe-Range Normal Form is defined recursively:

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$$\mathrm{rr}(R(t_1, \ldots, t_n)) = \{x \mid x \text{ is a variable among the } t_1, \ldots, t_n\}$$

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Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{\text{SID}}, y_{\text{Stop}}, y_{\text{To}}. \big(\text{Stops}(y_{\text{SID}}, y_{\text{Stop}}, \texttt{"true"}) \land \text{Connect}(y_{\text{SID}}, y_{\text{To}}, x_{\text{Line}})\big)[x_{\text{Line}}]$
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Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

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An FO query $q = \varphi[x_1, \ldots, x_n]$ is a safe-range query if $\mathrm{rr}(\mathrm{SRNF}(\varphi)) = \{\, x_1, \ldots, x_n \,\}$.

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Solution.

$\text{rr}(\varphi_1) = \{ x_{\text{Line}} \}$ SR, DI

$\text{rr}(\varphi_2) = \emptyset$ not SR, not DI

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