EXERCISE 5

Science of Computational Logic

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Problem 5.1

In the lectures, $\approx_{\mathcal{E}}$ was defined to be the *least congruence relation generated by* $\mathcal{E}$. What does it mean?

Problem 5.2

Consider the set of clauses

$$\mathcal{F} = \{\ [p(f(Y)), q(Y), r(b)],\ [\neg p(b)],\ [\neg q(a)],\ [\neg r(a)]\ \}$$

and the equational system

$$\mathcal{E} = \{(\forall X) f(X) \approx X, a \approx b\}.$$

Show by paramodulation, resolution and factoring that $\mathcal{F} \cup \mathcal{E} \cup \mathcal{E}_{\approx}$ is unsatisfiable. Also give the mgu θ used in every step.

Problem 5.3

Let $\mathcal{R}$ be a term rewriting system and let s and t be terms. Prove that:

1. $s \rightarrow_{\mathcal{R}} t$ implies $s \approx_{\mathcal{E}_{\mathcal{R}}} t$.
2. $s \leftrightarrow^*_{\mathcal{R}} t$ implies $s \approx_{\mathcal{E}_{\mathcal{R}}} t$.

Problem 5.4

A non terminating term rewriting system can be confluent. True or false? Prove it.

Problem 5.5

Prove that a term rewriting system $\mathcal{R}$ is Church-Rosser if and only if it is confluent.

Problem 5.6

Consider the following term rewriting system:

$$\begin{aligned} &f(f(X,Y),Z) \to f(X,f(Y,Z)); \\ &f(X,1) \to X. \end{aligned}$$

1. Is it terminating? Justify your answer.
2. Compute all the critical pairs, and show how you got them.
3. Can you orientate the critical pairs, i.e., add a rule $s \to t$ or $t \to s$ for each critical pair $\langle s,t \rangle$, such that termination is preserved? (If it is possible, do it . . .)

Note: When executing the completion algorithm you have to go on trying to build critical pairs with the iteratively added rules.

Problem 5.7

Let $\mathcal{R}$ be a term rewriting system and $>/2$ a termination ordering.

If for all rules $l \to r \in \mathcal{R}$ the relation $l > r$ holds, then $\mathcal{R}$ is terminating.

Problem 5.8

Consider the term rewriting system

$$\mathcal{R} = \{\, f(g(X)) \to g(X), \quad (1)$$

$$g(h(X)) \to g(X) \,\} \quad (2)$$

Show that $\mathcal{R}$ is canonical.