TECHNISCHE UNIVERSITÄT DRESDEN

# DATABASE THEORY

## Lecture 7: Query Optimisation

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TU Dresden, 26 May 2016

# Overview



See course homepage [⇒ link] for more information and materials



# Review

We have studied FO queries and the simpler conjunctive queries

Our focus was on query answering complexity:

| | Combined complexity | Query complexity | Data complexity |
|---|---|---|---|
| FO queries | $\mathrm{PSPACE}$-comp. | $\mathrm{PSPACE}$-comp. | in $\mathrm{AC}^0$ |
| Conjunctive queries | $\mathrm{NP}$-comp. | $\mathrm{NP}$-comp. | in $\mathrm{AC}^0$ |
| Tree CQs | in $\mathrm{P}$ | in $\mathrm{P}$ | in $\mathrm{AC}^0$ |
| Bound. Treewidth CQs | in $\mathrm{P}$ | in $\mathrm{P}$ | in $\mathrm{AC}^0$ |
| Bound. Hypertree w CQs | in $\mathrm{P}$ | in $\mathrm{P}$ | in $\mathrm{AC}^0$ |



# Static Query Optimisation

Can we optimise query execution without looking at the database?

Queries are logical formulae, so some things might follow . . .

Query equivalence: Will the queries $Q_1$ and $Q_2$ return the same answers over any database?

- In symbols: $Q_1 \equiv Q_2$
- We have seen many examples of equivalent transformations in exercises
- Several uses for optimisation:
  $\leadsto$ DBMS could run the "nicer" of two equivalent queries
  $\leadsto$ DBMS could use cached results of one query for the other
  $\leadsto$ Also applicable to equivalent subqueries

## Static Query Optimisation (2)

Other things that could be useful:

- Query emptiness: Will query $Q$ never have any results?

  $\rightsquigarrow$ Special equivalence with an "empty query"
  (e.g., $x \neq x$ or $R(x) \wedge \neg R(x)$)
  $\rightsquigarrow$ Empty (sub)queries could be answered immediately
- Query containment: Will the query $Q_1$ return a subset of the results of query $Q_2$? (in symbols: $Q_1 \sqsubseteq Q_2$)

  $\rightsquigarrow$ Generalisation of equivalence:
  $Q_1 \equiv Q_2$ if and only if $Q_1 \sqsubseteq Q_2$ and $Q_2 \sqsubseteq Q_1$
- Query minimisation: Given a query $Q$, can we find an equivalent query $Q'$ that is "as simple as possible."



## First-order logic: Decidable or not?

We have seen in recent lectures:

- FO queries can be answered in PSPACE (combined complexity) and $\mathrm{AC}^0$ (data complexity)
- FO queries correspond to relational algebra, so every relational DBMS answers FO queries in practice

In foundational courses on logic, you should have learned

- Reasoning in first-order logic is undecidable

Indeed, Wikipedia says it too (so it must be true . . . ):

- "Unlike propositional logic, first-order logic is undecidable (although semidecidable)" [Wikidedia article First-order logic]

Is the first-order logic we use different from the first-order logic used elsewhere? Is mathematics inconsistent?



## Solving the Mystery

All of the above are true for first-order logic
but people are studying different decision problems:

### Problem 1: Model Checking

- Given: a logical sentence $\varphi$ and a finite model $\mathcal{I}$
- Question: is $\mathcal{I}$ a model for $\varphi$, i.e., is $\varphi$ satisfied in $\mathcal{I}$?
- Corresponds to Boolean query entailment
- PSPACE-complete for first-order sentences

### Problem 2: Satisfiability Checking

- Given: a logical sentence $\varphi$
- Question: does $\varphi$ have any model?
- Equivalent to many reasoning problems (entailment, tautology, unsatisfiability, etc.)
- undecidable for first-order sentences



## Back to Query Optimisation

What do these results mean for query optimisation?

Two similar questions:

(1) Are the Boolean FO queries $\varphi_1$ and $\varphi_2$ equivalent?

(2) Are the FO sentences $\varphi_1$ and $\varphi_2$ equivalent?

$\rightsquigarrow$ So FO query equivalence is undecidable?

However, (1) is not equivalent to (2) but to the following:

(2') Are the FO sentences $\varphi_1$ and $\varphi_2$ equivalent in all finite interpretations?

$\rightsquigarrow$ finite-model reasoning for FO logic

# Finite-Model Reasoning

Does it really make a difference?

Yes. Example formula $\varphi$:

$$
\begin{aligned}
(\forall x.\exists y.R(x,y)) \wedge & \\
(\forall x, y_1, y_2.R(x,y_1) \wedge R(x,y_2) \to y_1 \approx y_2) \wedge & \qquad R \text{ is a function } \ldots \\
(\forall x_1, x_2, y.R(x_1,y) \wedge R(x_2,y) \to x_1 \approx x_2) \wedge & \qquad \ldots \text{ and injective } \ldots \\
(\exists y.\forall x.\neg R(x,y)) & \qquad \ldots \text{ but not surjective}
\end{aligned}
$$

Such a function $R$ can only exist over an infinite domain.
$\leadsto$ over finite models, $\varphi$ is unsatisfiable
$\leadsto$ $\varphi$ is finitely equivalent to $\forall x.R(x,x) \wedge \neg R(x,x)$
$\leadsto$ this equivalence does not hold on arbitrary models



# Trakhtenbrot's Theorem

Is finite-model reasoning easier than FO reasoning in general?

Unfortunately no:

**Theorem (Boris Trakhtenbrot, 1950)**
Finite-model reasoning of first-order logic is undecidable.

Interesting observation:

- The set of all true sentences (tautologies) of FO is recursively enumerable ("FO is semi-decidable")
- but the set of all FO tautologies under finite models is not.

$\leadsto$ finite model reasoning is harder than FO reasoning in this case!



# Let's Prove Trakhtenbrot's Theorem

Proof idea: reduce the Halting Problem to finite satisfiability

- Input of the reduction:
  a deterministic Turing Machine (DTM) $\mathcal{M}$ and an input string $w$
- Output of the reduction: a first-order formula $\varphi_{\mathcal{M},w}$
- Such that $\mathcal{M}$ halts on $w$ if and only if $\varphi_{\mathcal{M},w}$ has a finite model

Ok, this would do, because Halting of DTMs is undecidable
but how should we achieve this?

- Capture the computation of the DTM in a finite model
- The model contains the whole run: the tape and state for every computation step
- A finite part of the tape is enough if the DTM halts



# TM Runs as Finite Models

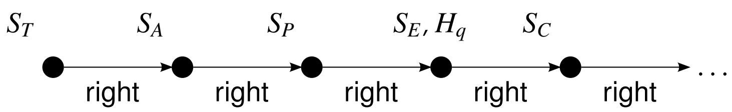
Recall: Turing Machine is given as $\mathcal{M} = \langle Q, q_{\text{start}}, q_{\text{acc}}, \Sigma, \Delta \rangle$
(state set $Q$, tape alphabet $\Sigma$ with blank $\llcorner\!\lrcorner$, transitions $\Delta \subseteq (Q \times \Sigma) \times (Q \times \Sigma \times \{l, r, s\})$)

A configuration is a (finite piece of) tape + a position + a state:

$q \in Q$
$\downarrow$

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | A | P | E | C | O | N | T | E | N | T | S | ␣ | … |

Here is how we want part of our model (database) to look:



# Encoding TM Runs as Relational Structures

We use several unary predicate symbols to mark tape cells:

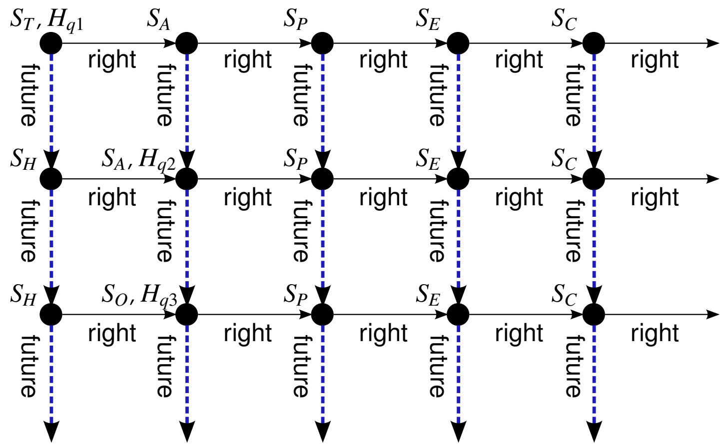
- $S_\sigma(\cdot)$ for each $\sigma \in \Sigma$: tape cell contains symbol $\sigma$
- $H_q(\cdot)$ for each $q \in Q$: head is at tape cell, and TM is in state $q$

We use two binary predicate symbols to connect tape positions:

- right$(\cdot,\cdot)$: neighbouring tape cells at same step
- right$^+(\cdot,\cdot)$: transitive super-relation of right
- future$(\cdot,\cdot)$: tape cells at same position in consecutive steps



# Intended Database




(right$^+$ is not shown)

We now need to specify formulae to enforce this intended structure (or something that is close enough to it).



# Defining the Initial Configuration

Require that right$^+$ is a transitive super-relation of right:

$$\begin{aligned}\varphi_{\text{right}^+} = {} & \forall x, y.(\text{right}(x,y) \to \text{right}^+(x,y)) \wedge \\ & \forall x, y, z.(\text{right}(x,y) \wedge \text{right}^+(y,z) \to \text{right}^+(x,z))\end{aligned}$$

Define start configuration for an input word $w = \sigma_1\sigma_2\ldots\sigma_n$:

$$\begin{aligned}\varphi_w = {} & \exists x_1,\ldots,x_n.H_{q_\text{start}}(x_1) \wedge \neg\exists z.\text{right}(z,x_1) \wedge \\ & S_{\sigma_1}(x_1) \wedge \neg\exists z.\text{future}(z,x_1) \wedge \text{right}(x_1,x_2) \wedge \\ & S_{\sigma_2}(x_2) \wedge \neg\exists z.\text{future}(z,x_2) \wedge \text{right}(x_2,x_3) \wedge \\ & \ldots \\ & S_{\sigma_n}(x_n) \wedge \neg\exists z.\text{future}(z,x_n) \wedge \\ & \forall y.(\text{right}^+(x_n,y) \to (S_\llcorner(y) \wedge \neg\exists z.\text{future}(z,y)))\end{aligned}$$

$\leadsto$ there can be any number of cells right of the input, but they must contain $\llcorner$.



# Consistent Tape Contents, Head, and State

A cell can only contain one symbol:

$$\varphi_S = \bigwedge_{\sigma,\sigma'\in\Sigma,\sigma\neq\sigma'} \forall x.(\neg S_\sigma(x) \vee \neg S_{\sigma'}(x))$$

The TM is never at more than one position:

$$\begin{aligned}\varphi_H = {} & \bigwedge_{q\in Q} \forall x,y.\left(H_q(x) \wedge \text{right}^+(x,y) \to \bigwedge_{q'\in Q} \neg H_{q'}(y)\right) \wedge \\ & \bigwedge_{q\in Q} \forall x,y.\left(\text{right}^+(x,y) \wedge H_q(y) \to \bigwedge_{q'\in Q} \neg H_{q'}(x)\right)\end{aligned}$$

The TM can only be in one state:

$$\varphi_Q = \bigwedge_{q,q'\in Q,q\neq q'} \forall x.(\neg H_q(x) \vee \neg H_{q'}(x))$$

## Transitions

For every non-moving transition $\delta = \langle q, \sigma, q', \sigma', s\rangle \in \Delta$:

$$\varphi_\delta = \forall x.H_q(x) \wedge S_\sigma(x) \to \exists y.\mathsf{future}(x, y) \wedge S_{\sigma'}(y) \wedge H_{q'}(y)$$

For every right-moving transition $\delta = \langle q, \sigma, q', \sigma', r\rangle \in \Delta$:

$$\varphi_\delta = \forall x.H_q(x) \wedge S_\sigma(x) \to \exists y.\mathsf{future}(x, y) \wedge S_{\sigma'}(y) \wedge \exists z.\mathsf{right}(y, z) \wedge H_{q'}(z)$$

For every left-moving transition $\delta = \langle q, \sigma, q', \sigma', l\rangle \in \Delta$:

$$\varphi_\delta = \forall x.H_q(x) \wedge S_\sigma(x) \wedge (\exists v.\mathsf{right}(v, x)) \to \exists y.\mathsf{future}(x, y) \wedge S_{\sigma'}(y) \wedge \exists z.\mathsf{right}(y, z) \wedge H_{q'}(z)$$

Summing all up:

$$\varphi_\Delta = \bigwedge_{\delta\in\Delta} \varphi_\delta$$



## Preserve Tape if not Changed by Transition

Contents of tape cells that are not under the head are kept:

$$\varphi_{\mathsf{mem}} = \forall x, y. \bigwedge_{\sigma\in\Sigma} \left( S_\sigma(x) \wedge \left( \bigwedge_{q\in Q} \neg H_q(x) \right) \wedge \mathsf{future}(x, y) \to S_\sigma(y) \right)$$



## Building the Configuration Grid

If one cell has a future ($\to$) or past ($\leftarrow$), respectively, all cells of the tape do:

$$\varphi_{fp1} = \forall x_2, y_1.(\exists x_1.\mathsf{right}(x_1, y_1) \wedge \mathsf{future}(x_1, x_2)) \leftrightarrow (\exists y_2.\mathsf{future}(y_1, y_2) \wedge \mathsf{right}(x_2, y_2))$$
$$\varphi_{fp2} = \forall x_1, y_2.(\exists y_1.\mathsf{right}(x_1, y_1) \wedge \mathsf{future}(y_1, y_2)) \leftrightarrow (\exists x_2.\mathsf{future}(x_1, x_2) \wedge \mathsf{right}(x_2, y_2))$$

Left ($l$) and right ($r$) neighbours, and future ($f$) and past ($p$) are unique:

$$\varphi_r = \forall x, y, y'.\mathsf{right}(x, y) \wedge \mathsf{right}(x, y') \to y \approx y'$$
$$\varphi_l = \forall x, x', y.\mathsf{right}(x, y) \wedge \mathsf{right}(x', y) \to x \approx x'$$
$$\varphi_f = \forall x, y, y'.\mathsf{future}(x, y) \wedge \mathsf{future}(x, y') \to y \approx y'$$
$$\varphi_p = \forall x, x', y.\mathsf{future}(x, y) \wedge \mathsf{future}(x', y) \to x \approx x'$$



## Finishing the Proof of Trakhtenbrot's Theorem

We obtain a final FO formula

$$\varphi_{\mathcal{M},w} = \varphi_{\mathsf{right}^+} \wedge \varphi_w \wedge \varphi_S \wedge \varphi_H \wedge \varphi_Q \wedge \varphi_\Delta \wedge \varphi_{\mathsf{mem}} \wedge \varphi_{fp1} \wedge \varphi_{fp2} \wedge \varphi_r \wedge \varphi_l \wedge \varphi_f \wedge \varphi_p$$

Then $\varphi_{\mathcal{M},w}$ is finitely satisfiable if and only if $\mathcal{M}$ halts on $w$:

- If $\mathcal{M}$ has a finite run when started on $w$ then $\varphi_{\mathcal{M},w}$ has a finite model that encodes this run.
- If $\varphi_{\mathcal{M},w}$ has a finite model, then we can extract from this model a finite run of $\mathcal{M}$ on $w$.

Note: the proof can be made to work using only one binary relation symbol and no equality (not too hard, but less readable)

## The Impossibility of FO Query Optimisation

Trakhtenbrot's Theorem has severe consequences for static FO query optimisation

All of the following decision problems are undecidable (exercise):

- Query equivalence
- Query emptiness
- Query containment

$\rightsquigarrow$ "perfect" FO query optimisation is impossible

Other important questions about FO queries are also undecidable, for example:

- Is a given FO query domain independent?



## Is Query Optimisation Futile?

Not quite: things are simpler for conjunctive queries

Conjunctive query containment – example:

$$Q_1 : \quad \exists x, y, z.\ R(x, y) \wedge R(y, y) \wedge R(y, z)$$
$$Q_2 : \quad \exists u, v, w, t.\ R(u, v) \wedge R(v, w) \wedge R(w, t)$$

$Q_1$ find $R$-paths of length two with a loop in the middle
$Q_2$ find $R$-paths of length three

$\rightsquigarrow$ in a loop one can find paths of any length
$\rightsquigarrow$ $Q_1 \sqsubseteq Q_2$



## Summary and Outlook

There are many well-defined static optimisation tasks that are independent of the database
$\rightsquigarrow$ query equivalence, containment, emptiness

Unfortunately, all of them are undecidable for FO queries $\rightsquigarrow$
Slogan: "all interesting questions about FO queries are undecidable"
Next topics:

- More positive results for conjunctive queries
- Measure expressivity rather than just complexity
- Look at query languages beyond first-order logic