Simulating Sets in Answer Set Programming

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Sarah Alice Gaggl, Philipp Hanisch, Markus Krötzsch

Department of Computer Science
TU Dresden

TECHNISCHE UNIVERSITÄT DRESDEN

International Center for Computational Logic

Answer Set Programming with Sets

Approach: Extend ASP with terms that represent finite sets of domain elements.

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Example: Defining connected components

$$edge^+(X,Y) \leftarrow edge(X,Y)$$
$$edge^+(X,Y) \leftarrow edge^+(X,Z) \wedge edge(Z,Y)$$

$$comp(\{X\}) \leftarrow vertex(X)$$
$$comp(S \cup \{Y\}) \leftarrow comp(S) \wedge X \in S \wedge edge^+(X,Y) \wedge edge^+(Y,X)$$

$$subComp(S_1) \leftarrow comp(S_1) \wedge comp(S_2) \wedge S_1 \subseteq S_2 \wedge \textbf{not}\ S_2 \subseteq S_1$$
$$maxComp(S) \leftarrow comp(S) \wedge \textbf{not}\ subComp(S)$$

Reasoning with DLP(S)

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These bounds are tight:

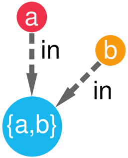
Theorem: Complexity of deciding fact entailment under stable-model semantics:

DLP(S)	coNExpTime$^{\text{NP}}$-complete
DLP(S) without $\vee$	coNExpTime-complete

Hardness holds even if only facts are allowed to vary (data complexity).

Simulating Sets with Function Symbols and Negation

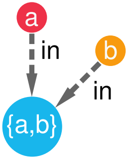
Sets can be encoded in facts:



$in(X, S)$: "$X \in S$"

Simulating Sets with Function Symbols and Negation

Function terms can represent sets:



$f_\cup(\mathrm{b}, f_\cup(\mathrm{a}, \mathrm{c}_\varnothing))$

$in(X, S)$: "$X \in S$"

$f_\cup(X, S)$: "$\{X\} \cup S$"

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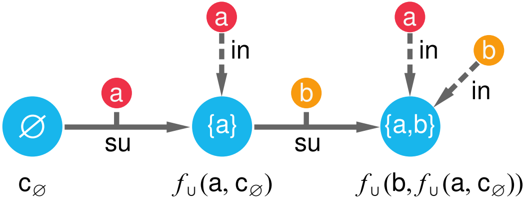
Simulating Sets with Function Symbols and Negation

Sets can be constructed element-by-element:



$in(X, S)$: "$X \in S$"

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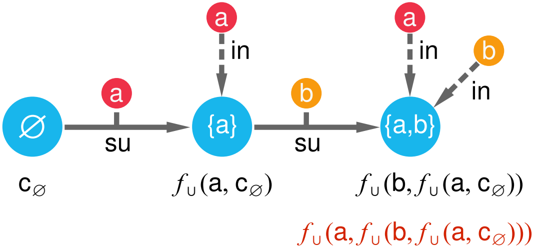
$su(X, S, T)$: "$\{X\} \cup S = T$"

Simulating Sets with Function Symbols and Negation



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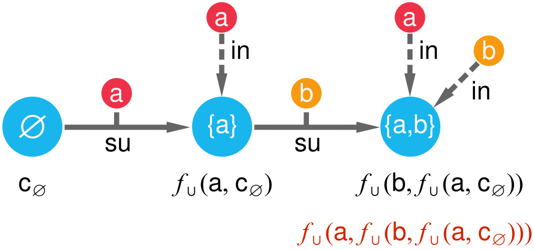

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Problem: Duplicate elements in terms $\rightsquigarrow$ infinitely many terms for finitely many sets

Solution: Use negation to prevent duplicates:

$$su(X, S, f_\cup(X, S)) \leftarrow get\_su(X, S) \wedge \textbf{not}\ in(X, S)$$

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Solution 1: Use lazy grounders

- Lazy ASP systems compute grounding on-demand
  $\leadsto$ non-stratified negations not ignored during grounding
- Systems like **Alpha** [Weinzierl et al.] can produce finite stable models

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- If $in(X, S)$ holds, then also $su(X, S, S)$, and therefore $\exists V.su(X, S, V)$
  $\rightsquigarrow$ standard redundancy check in existential rule reasoning mimics negation
- Use **existential rule reasoners for grounding**

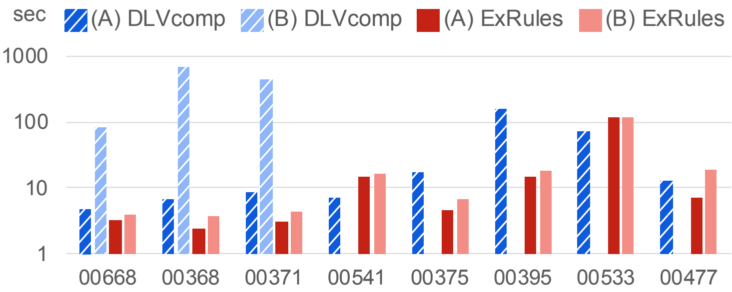
Evaluation: Setup

Use ASP to solve non-monotonic, ExpTime-complete problems for OWL ontologies

- Implement an ontology reasoner in DLP(S)
- Add additional non-monotonic rules to
  (A) compute the reduced class hierarchy
  (B) compute maximal antichains in the class hierarchy
- Evaluated implementations:
  Alpha (lazy grounder + function-term rewriting)
  ExRules (VLog existential rule grounder + clasp solver)
  DLVcomplex (native DLP(S) implementation)
- Test data:
  8 real-world ontologies, entailing 11K–911K subclass relations

Evaluation: Results

- Alpha could not solve task (A) or (B) in any case, but it could compute plain OWL inferences for one ontology
- DLVcomplex could solve (A) in all cases, and (B) for 3 (of 8)
- ExRules could solve (A) and (B) in all cases



⇝ Initial feasibility study – should not exclude any implementation approach yet

Conclusion

We extend disjunctive logic programming with set-terms, study its complexity, and show that it can solve reasoning problems beyond the propositional case.

Main contributions:

- Complexity results for stable-model reasoning with set terms
- Simulation of sets in logic programs with functions **and** finite stable models
- Set-aware grounding using existential rule reasoning
- New grounding implementation and evaluation

Open questions:

- Could lazy grounding approaches perform better here?
- In general: how to make set-related reasoning performance more robust?
- Further use cases to exploit the added expressivity?

Rewatch talk: `https://tinyurl.com/asp-sets-22` IJCAI'22 Poster #174