TU Dresden, Fakultät Informatik
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Complexity Theory
**Exercise 5: Space Complexity**
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**Exercise 5.1.** Consider the Japanese game *go-moku* that is played by two players X and O on a 19x19 board. Players alternately place markers on the board, and the first one to have five of their markers consecutively in a row, column, or diagonal wins.

Consider the generalized version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game, together with a marker which player moves next. We define

$$\mathbf{GM} = \{\langle B\rangle \mid B \text{ is a position of go-moku where X has a winning strategy}\}.$$

Show that **GM** is in $\mathrm{PS}\textsc{pace}$.

**Exercise 5.2.** Show that the universality problem of nondeterministic finite automata

$$\mathbf{ALL}_{\text{NFA}} = \{\langle \mathcal{A}\rangle \mid \mathcal{A} \text{ an NFA accepting every valid input}\}$$

is in $\mathrm{PS}\textsc{pace}$.

**Hint:**
Prove that, if $\mathbf{L}(\mathcal{A}) \neq \Sigma^*$ and $\mathcal{A}$ has $n$ states, then there exists a word $w \in \Sigma^*$ of length at most $2^n$ such that $w \notin \mathbf{L}(\mathcal{A})$. Then, use this fact to give a non-deterministic algorithm whose space consumption is polynomially bounded. Finally, apply Savitch's Theorem.

**Exercise 5.3.** Consider the following problem: given a string of parentheses, decide if they are properly matched and nested, (e.g., (()())() satisfy the requirement, but ()) or )( do not). Show that this problem is in L.

**Exercise 5.4.** Is it true that **Clique** $\leq_L$ **IndependentSet**?

**Exercise 5.5.** The class NL consists of all problems solvable by nondeterministic log-space Turing machines. Can you give an equivalent verifier-based characterization, analogous to the standard verifier-based definition of NP?

**Exercise 5.6.** Let $A, B \subseteq \{0,1\}^*$ and $\varnothing \neq B \neq \{0,1\}^*$. Prove that, if $A \in \mathrm{NL}$ and $B \leq_L A$, then $B \leq_p$ **Palindromes**.

**Palindromes** = $\{w \mid w \in \{0,1\}^*$ is a palindrome (reads the same from left to right and from right to left)$\}$

**Exercise 5.7.** Show that the composition of log-space reductions yields a log-space reduction.

**Exercise 5.8.** Show that the word problem $\mathsf{A}_{\mathsf{NFA}}$ of non-deterministic finite automata is $\mathsf{NL}$-complete.

**Exercise 5.9.** Show that

$$\mathsf{BIPARTITE} = \{\, \langle G \rangle \mid G \text{ a finite bipartite graph} \,\}$$

is in $\mathsf{NL}$. For this show that $\overline{\mathsf{BIPARTITE}} \in \mathsf{NL}$ and use $\mathrm{NL} = \mathrm{coNL}$.

**Hint:**
Show that a graph $G$ is bipartite if and only if it does not contain a cycle of odd length.