Problem 8.1

1. Consider the following definite programs \( P_1 \) and \( P_2 \):

\[
P_1 = \{ \begin{array}{l}
even(0) \leftarrow \top, \\
odd(s(0)) \leftarrow \top, \\
\even(s(X)) \leftarrow \odd(X), \\
odd(s(X)) \leftarrow \even(X) \\
\end{array} \}
\]

\[
P_2 = \{ \begin{array}{l}
even(0) \leftarrow \top, \\
\even(s(s(0))) \leftarrow \even(X), \\
\even(X) \leftarrow \even(s(s(X))) \\
\end{array} \}
\]

Which interpretation is a two-valued model of \( P_1 \) and \( P_2 \)?

2. Consider the following definition for definite programs:

**Acceptable Definite Program** Let \( P \) be a definite program, \( \ell \) a level mapping for \( P \) and \( \mathcal{M} \) a two-valued model of \( P \). \( P \) is called *acceptable with respect to \( \ell \) and \( \mathcal{M} \)* if for every clause \( A \leftarrow B_1, \ldots, B_m \in gP \) the following implication holds for all \( i \) with \( 1 \leq i \leq m \):

\[
\mathcal{M}(B_1 \land \cdots \land B_{i-1}) = \top \text{ implies } \ell(A) > \ell(B_i).
\]

\( P \) is called *acceptable* if it is acceptable with respect to some level mapping and some model of \( P \).

3. Are \( P_1 \) and \( P_2 \) acceptable programs? Motivate your answer.

Problem 8.2

Show that the following proposition holds:

**Proposition 19** Let \( P \) be a program, \( \ell \) a (total) level mapping for \( P \), \( \mathcal{I} \) the set of (three-valued) interpretations for \( P \), and \( I, J \in \mathcal{I} \). The function \( d_\ell : \mathcal{I} \times \mathcal{I} \rightarrow \mathbb{N} \) defined as

\[
d_\ell(I, J) = \begin{cases} 
(\frac{1}{2})^n & I \neq J \text{ and } I(A) = J(A) \neq U \text{ for all } A \text{ with } \ell(A) < n \\
0 & I(A) \neq J(A) \text{ or } I(A) = J(A) = U \text{ for some } A \text{ with } \ell(A) = n 
\end{cases}
\]

is a metric.

Problem 8.3

Show that the deduction theorem is not satisfied under Łukasiewicz and Kleene logic.

Problem 8.4

Show that the following lemma holds:

**Lemma 22** Let \( I \) be the least fixed point of \( \Phi_P \) and \( J \) be a model of \( wcP \). Then for every ground atom \( A \), the following holds:

(1) If \( I(A) = \top \) then \( J(A) = \top \) and (2) If \( I(A) = \bot \) then \( J(A) = \bot \).