EXERCISE 8

Human Reasoning and Computational Logic

Steffen Hölldobler, Emmanuelle Dietz

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Problem 8.1

1. Consider the following definite programs $\mathcal{P}_1$ and $\mathcal{P}_2$:

$$\begin{array}{rlrcl}
\mathcal{P}_1 & = \{ & even(0) & \leftarrow & \top, \\
& & odd(s(0)) & \leftarrow & \top, \\
& & even(s(X)) & \leftarrow & odd(X), \\
& & odd(s(X)) & \leftarrow & even(X) \quad \} \\
\mathcal{P}_2 & = \{ & even(0) & \leftarrow & \top, \\
& & even(s(s(0))) & \leftarrow & even(X), \\
& & even(X) & \leftarrow & even(s(s(X))) \quad \}
\end{array}$$

   Which interpretation is a two-valued model of $\mathcal{P}_1$ and $\mathcal{P}_2$?

2. Consider the following definition for definite programs:

   Acceptable Definite Program Let $\mathcal{P}$ be a definite program, ℓ a level mapping for $\mathcal{P}$ and $\mathcal{M}$ a two-valued model of $\mathcal{P}$. $\mathcal{P}$ is called *acceptable with respect to* ℓ *and* $\mathcal{M}$ if for every clause $A \leftarrow B_1, \ldots, B_m \in g\mathcal{P}$ the following implication holds for all i with $1 \leq i \leq m$:

$$M(B_1 \wedge \cdots \wedge B_i - 1) = \top \text{ implies } \ell(A) > \ell(B_i).$$

   $\mathcal{P}$ is called *acceptable* if it is acceptable with respect to some level mapping and some model of $\mathcal{P}$

3. Are $\mathcal{P}_1$ and $\mathcal{P}_2$ acceptable programs? Motivate your answer.

Problem 8.2

Show that the following proposition holds:

Proposition 19 Let $\mathcal{P}$ be a program, ℓ a (total) level mapping for $\mathcal{P}$, $\mathcal{I}$ the set of (three-valued) interpretations for $\mathcal{P}$, and $I, J \in \mathcal{I}$. The function $d_\ell : \mathcal{I} \times \mathcal{I} \mapsto \mathbb{N}$ defined as

$$d_\ell(I,J) = \begin{cases} (\frac{1}{2})^n & I \neq J \text{ and } I(A) = J(A) \neq \mathsf{U} \text{ for all A with } \ell(A) < n \text{ and} \\ & I(A) \neq J(A) \text{ or } I(A) = J(A) = \mathsf{U} \text{ for some } A \text{ with } \ell(A) = n \\ 0 & \text{otherwise.} \end{cases}$$

is a metric.

Problem 8.3

Show that the deduction theorem is not satisfied under Łukasiewicz and Kleene logic.

Problem 8.4

Show that the following lemma holds:

Lemma 22 Let I be the least fixed point of $\Phi_{\mathcal{P}}$ and J be a model of wc$\mathcal{P}$. Then for every ground atom A, the following holds:

(1) If $I(A) = \top$ then $J(A) = \top$ and (2) If $I(A) = \bot$ then $J(A) = \bot$.