Finite and algorithmic model theory (Dresden, Winter 22/23): Exercises 2 (27.10.22 13:00)

1. Show that the random graph contains as a subgraph every finite graph.
2. Prove that $\mu_\infty(\sigma_{s,t}) = 1$ holds for all extension axioms $\sigma_{s,t}$.[1]
3. Show that the presence of constants in the signature spoils the 0–1 law of FO.
4. Show that the Second Order Logic does not have the 0–1 law.[2]
5. Alternative definition of the random graph.[3] Consider a countable graph G whose universe is the set of primes congruent to 1 modulo 4. Put an edge between p and q if p is a quadratic residue modulo q. Prove that G is isomorphic to the random graph.
6. Yet another alternative definition of the random graph.[4] We define the set HF of *heriditarily finite sets* as follows. The empty set $\emptyset$ is in HF and if $a_1, \ldots, a_k$ are in HF (for any $k \in \mathbb{N}$) then $\{a_1, \ldots, a_k\} \in \mathrm{HF}$. Consider a countable graph G whose domain is HF and we put the edge between two nodes u, v iff $u \in v$ or $v \in u$. Prove that G is isomorphic to the random graph.
7. Read Section 5.3 from [Grädel's notes] and sketch the proof that FO has 0–1 law for arbitrary purely relational symbols.
8. Prove that it is decidable to check for a given $\varphi \in \mathsf{FO}[\{\mathrm{E}\}]$ if φ is almost surely true.[5]

[1] Hint: Present the proof of Lemma 4.7 from [Grädel's notes]
[2] Hint: Express even!
[3] Difficult?
[4] Difficult?
[5] Employ the random graph and the fact that the random graph satisfies φ iff φ is almost surely true.

 Last edited on 19/10/2022 at 22:38.