TECHNISCHE UNIVERSITÄT DRESDEN

SECAI

TAMING DILATION IN IMPRECISE POOLING

Jonas Karge

Computational Logic Group

TU Dresden

Prima2024, Kyoto, November 22nd, 2024

Introduction

Suppose, we are dealing with

- a set of **agents** (people, sensors, drones, ...)
- that **vote** (via some voting rule)
- for **alternatives** (policies, sensor data, courses of action, ...).

Two distinct goals for voting procedures:

(1) Ensure a fair voting procedure;

(2) identify the correct alternative.

We assume: There is exactly one correct alternative, the **ground truth**.

Agents and Imprecise Beliefs

Imprecise Pooling

Scenario: Multiple experts assign imprecise probabilities, representing their belief, to an event such as:

> **Example:** Global sea level will rise at least 1.5 meters until the year 2100 above the level of 2000.

An imprecise probabilistic belief $[a, b]$ in a proposition A is denoted by $\mathcal{P}_1(A), ..., \mathcal{P}_n(A)$ for agents $a_1, ..., a_n$.

An imprecise pooling function takes as **input** n imprecise beliefs, one for each agent, for an event and yields as **output** a single collective imprecise belief.

Most basic pooling functions:

- Convex Pooling: Union of all input beliefs.
  Example: $Input : [0.1, 0.6], [0.5, 0.9]; Output : [0.1, 0.9]$.
- Pooling by Intersection: Intersection of all input beliefs.
  Example: $Input : [0.1, 0.6], [0.5, 0.9]; Output : [0.5, 0.6]$.

Dilation

- ”The specter [that] is haunting the theory of imprecise probabilities [is] the specter of dilation” .
- Dilation occurs when the acquisition of a new observation unavoidably leads to an increase in uncertainty (the aggregate interval becomes larger).
- Contraction occurs when the acquisition of a new observation unavoidably leads to a decrease in uncertainty (the aggregate interval becomes smaller).

Definition (Central Problem):
Given: $\mathcal{P}_1(A), ..., \mathcal{P}_n(A), \mathcal{P}_{n+1}(A)$ and $\mathcal{F}(\mathcal{P}_1(A), ..., \mathcal{P}_n(A))$.
Question: Under what conditions and to what extent should $\mathcal{F}(\mathcal{P}_1(A), ..., \mathcal{P}_{n+1}(A))$ dilate or contract compared to $\mathcal{F}(\mathcal{P}_1(A), ..., \mathcal{P}_n(A))$?

Approach used in our paper: Voting for Bins.

Voting and Voting for Bins

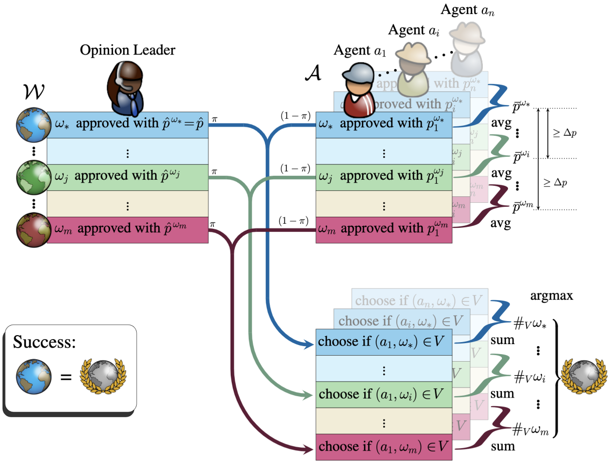
The Condorcet Jury Theorem (CJT)

Marie Jean Antoine Nicolas Caritat Marquis de Condorcet

Theorem: For odd-numbered **homogenous** groups of **independent** and **reliable** agents in a **dichotomic** voting setting, the probability that majority voting identifies the correct alternative

- increases monotonically with the number of agents and (non-asymptotic part)
- converges to 1 as the number of agents goes to infinity. (asymptotic part)

Voting



For this framework, we could

- prove the asymptotic part of the CJT;
- derive practical bounds on the number of agents necessary to identify the correct alternative with a given minimal success probability.

Probabilistic Belief Aggregation based on the CJT

Scenario: Multiple experts assess the likelihood of an event such as:

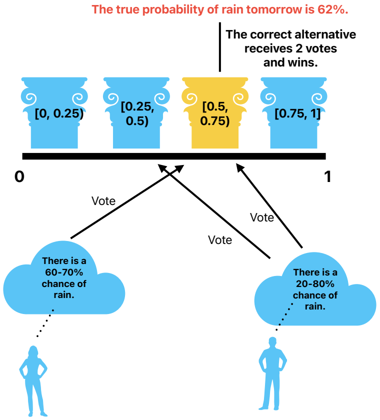
Example: There will be rain tomorrow in Dresden.

Two central assumptions that we make:

- There exists a **true probability** for an event to occur;
- the agent's beliefs are represented by **imprecise probabilities.**

From these assumptions, and exploiting the CJT, we defined an aggregation framework for imprecise probabilistic beliefs, referred to as **Voting for Bins.**

Aggregation Example



Example: Precision. Consider the following parameters: $P_{min} = 0.9, \Delta_p = 0.4$, $\hat{p} = 0.5, \pi = 0.1$, and $n = 200$. This yields a maximal number of 173 bins.

Alternatives and Dilation

Aggregation Example

Theorem: Consider an approval voting setting as described in the illustration earlier. Then, given a probability $P_{\min} < 1$, it is guaranteed that the success probability of the approval voting process is greater than $P_{\min}$ if the number m of bins is at most

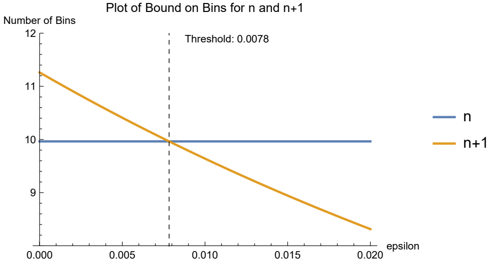
$$m \leq \frac{1-P_{min}}{(\hat{p}(e^{-\frac{1}{2}n(1-\pi)^2\Delta p^2})+(1-\hat{p})e^{-\frac{1}{2}n(\Delta p(1-\pi)-\pi)^2}} + 1.$$

Corollary 1

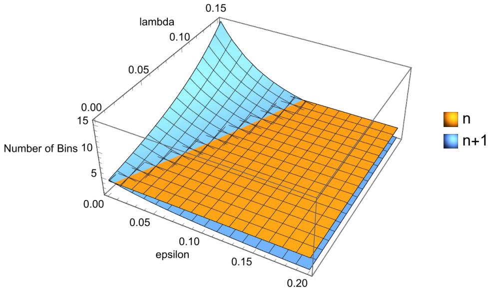
If agent a_{n+1} raises the average competence level / decreases average correlation, the maximal number of bins increases, resulting in a contraction of the pooled interval.

Corollary 2

If agent a_{n+1} increases the average correlation by an amount ε that outweighs just adding a new agent or if the agent increases the average competence that does not outweigh an increase of the average correlation, the aggregate dilates.



(a) Threshold for single increase in correlation.



(b) Increase in correlation and competence.

Number of Bins depending on ε and for $\Delta p = 0.6, P_{min} = 0.4, n = 20, \pi = 0.05$ (left), and depending on ε and λ for $P_{min} = 0.2, \Delta p = 0.4, n = 25, \pi = 0.05$ (right).

Future Work

Regarding the CJT:

- Translate the OL model into standard correlation measures,
- Derive bounds for liquid democracy (delegated voting).

Regarding VfB:

- Identify useful applications,
- Generalize VfB to the aggregation of probability distributions,
- Generalize VfB to the aggregation of (non-probabilistic) intervals.

Contact Information: jonas.karge@tu-dresden.de