EXERCISE 2

Human Reasoning and Computational Logic

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Problem 2.1

Given the first-order language $\mathcal{L}(\{p/1, q/1, r/0\}, \{a/0, b/0\}, \mathcal{V})$ and the following definite program $\mathcal{P}$:

$$p(a)$$
$$q(X) \leftarrow p(X)$$

Find all Herbrand models of $\mathcal{P}$. Which of them is the least?

Problem 2.2

Find out whether there exists a least Herbrand model for each subset of the following set

$$\mathcal{C} = \{[p(a), p(b)], [p(a)], [p(c)]\}$$

of clauses, and if yes, specify it.

Problem 2.3

Let $\mathcal{P}$ be a definite program and let I be a Herbrand interpretation of $\mathcal{P}$. Show that I is a model of $\mathcal{P}$ iff for every ground instance $A \leftarrow B_1 \wedge \ldots \wedge B_m$ of every clause in $\mathcal{P}$ the following holds: If $\{B_1, \ldots, B_m\} \subseteq I$ then $A \in I$.

Problem 2.4

Given the following program $\mathcal{P}$:

$$p(a, X, X)$$
$$p(s(X), Y, s(Z)) \leftarrow p(X, Y, Z)$$

1. Compute the answer substitutions for the queries

   (a) $\leftarrow p(s(a), X, s(s(a)))$.

   (b) $\leftarrow p(s(s(a)), X, s(s(s(a))))$.

   Compute in turn the occurring unification problems and the respective mgus.

2. Find the least Herbrand model of $\mathcal{P}$.

Problem 2.5

Given the definite program clauses

$r(a)$
$p(Y)$
$q(X) \leftarrow p(X)$
$s(X) \leftarrow q(X)$

and the definite goal $\leftarrow s(U)$.

1. Find an SLD refutation. (Indicate with every resovent which clauses have been resolved upon.)
2. Which answer substitution did you compute in the preceding subproblem?
3. Find a correct answer substitution for the definite program given above, which is different from the computed one.
4. Find a substitution which is no answer substitution for the definite program given above.

Problem 2.6

Given the following definite program $\mathcal{P}$:

$p(0, X, X)$
$p(s(X), Y, s(Z)) \leftarrow p(X, Y, Z)$
$r(s(0), X, X)$
$r(s(X), Y, Z) \leftarrow r(X, Y, W), p(W, Y, Z)$

Find a SLD derivation for the definite goal $\leftarrow r(s(s(0)), s(0), X)$.
What is the computed answer substitution?

Problem 2.7

Prove or refute the following assertions.

1. A definite program which consists just of facts
   never computes a nonempty answer substitution.
2. Every propositional definite program $\mathcal{P}$ (resp. every first-order definite program in which occur only 0-ary predicate symbols) is satisfiable.
3. A correct answer substitution is always a ground substitution.