EXERCISE 3

Human Reasoning and Computational Logic

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Problem 3.1

Consider program $\mathcal{P}$ consisting of the following three clauses:

$$\begin{array}{lcl} p(X) & \leftarrow & \neg q(X) \wedge r(X) \wedge t(X). \\ p(X) & \leftarrow & \neg s(X) \wedge r(X). \\ t(a) & \leftarrow & \top. \end{array}$$

Assume that $\mathcal{IC} = \emptyset$ and that $\mathcal{O} = \{p(a)\}$, and that the set of abducibles $\mathcal{A}_\mathcal{P}$ consists of the following facts and assumptions:

$$\begin{array}{lclclclclcl} q(a) & \leftarrow & \top. & \quad & r(a) & \leftarrow & \top. & \quad & s(a) & \leftarrow & \top. \\ q(a) & \leftarrow & \bot. & \quad & r(a) & \leftarrow & \bot. & \quad & s(a) & \leftarrow & \bot. \end{array}$$

1. What are the (minimal) explanations for $\mathcal{O}$ given $\mathcal{P}$?
2. What follows skeptically and credulously from $\mathcal{P}$ and $\mathcal{O}$?

Problem 3.2

Show that the following proposition holds:

Proposition Let $\mathcal{P}$ be a propositional logic program. Computing the least model of wc $\mathcal{P}$ under the Łukasiewicz logic can be done in polynomial time.

Problem 3.3

Consider the following proposition:

Proposition Let $\langle \mathcal{P}, \mathcal{A}, \mathcal{IC}, \models_{wcs} \rangle$ be an abductive framework, where $\mathcal{P}$ is a propositional logic program. Deciding whether $\mathcal{E}$ is an explanation for $\mathcal{O}$ given $\mathcal{P}$ can be done in polynomial time.

Show that the proposition holds by showing the following:

1. $\mathcal{E}$ is a consistent subset of $\mathcal{A}$,
2. wc $(\mathcal{P} \cup \mathcal{E})$ is consistent under Łukasiewicz logic and
3. $\mathcal{P} \cup \mathcal{E} \models_{wcs} \mathcal{O}$.

Problem 3.4

Show that the following proposition holds

Proposition Let $\langle \mathcal{P}, \mathcal{A}, \mathcal{IC}, \models_{wcs} \rangle$ be an abductive framework, where $\mathcal{P}$ is a propositional logic program. Deciding, whether $\mathcal{E}$ is a minimal explanation of $\mathcal{O}$ can be done in polynomial time.