The Chase in Lean

Crafting a Formal Library for Existential Rule Research

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Knowledge-Based Systems Group, TU Dresden, Germany

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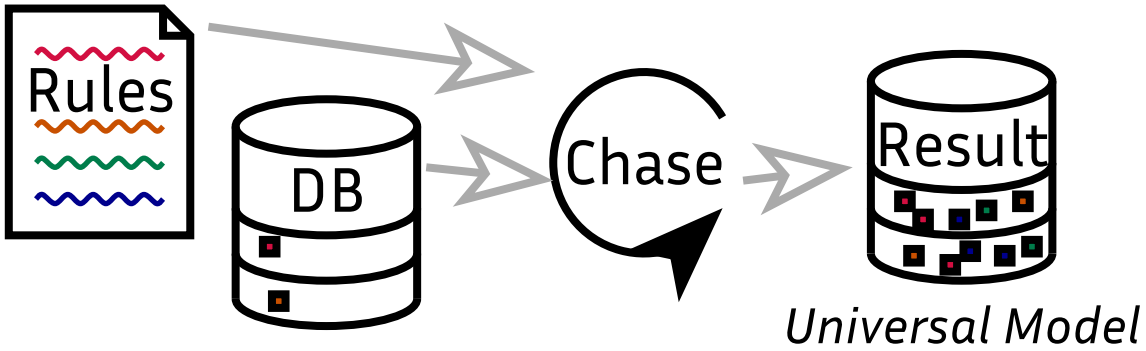
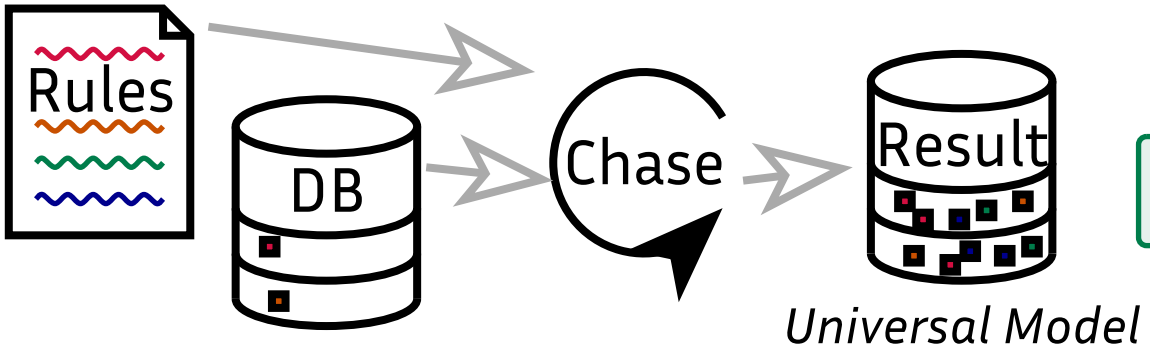
Chase Crash Course



Chase Crash Course



Universal Model

$$\mathrm{R}(x, y) \to \exists z.\ \mathrm{R}(y, z) \wedge \mathrm{R}(z, y)$$

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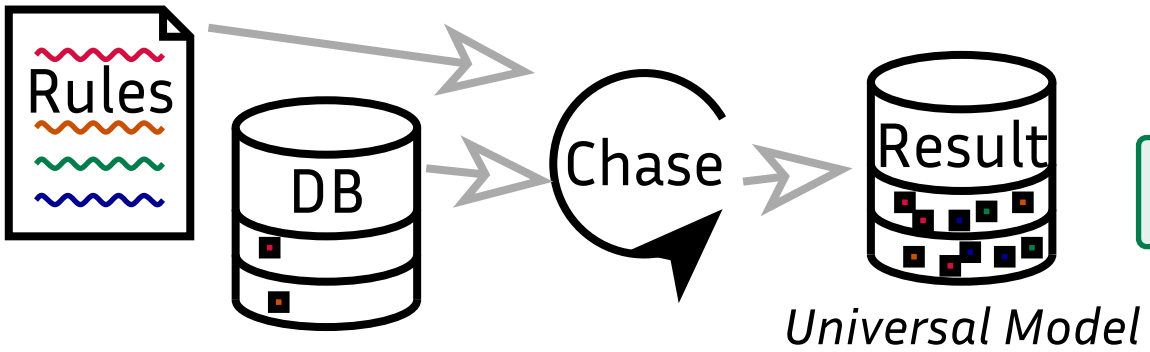

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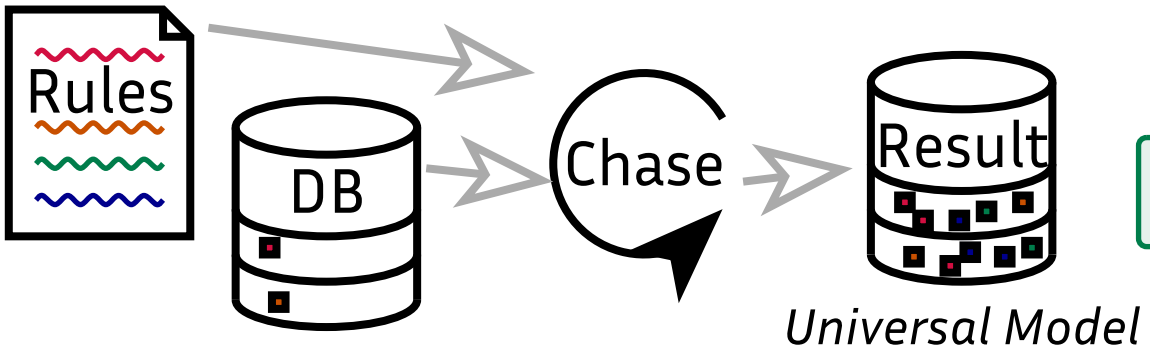
DB: $\mathrm{R}(c, d)$

Chase Crash Course



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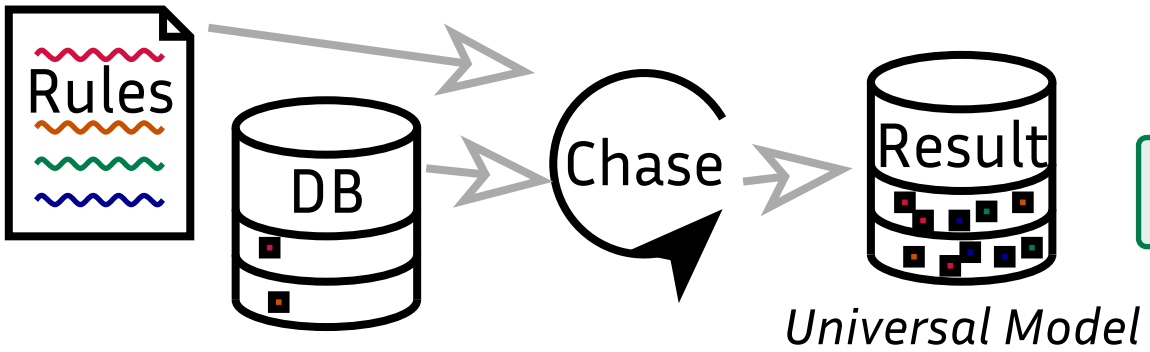

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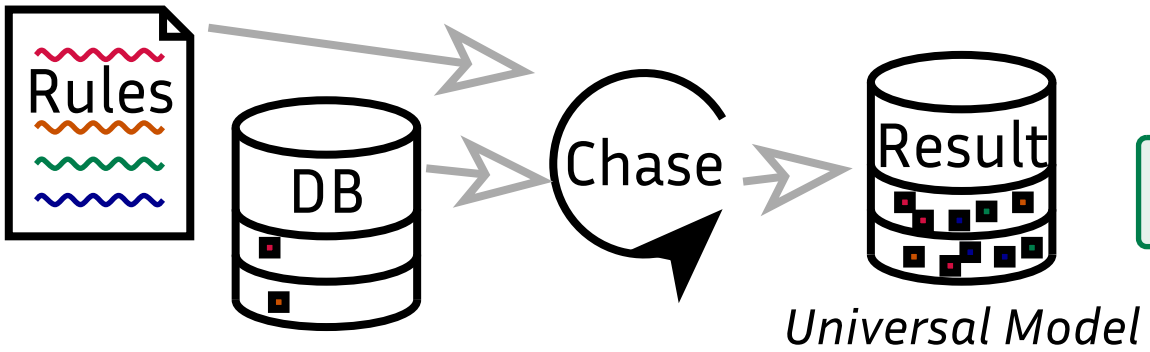

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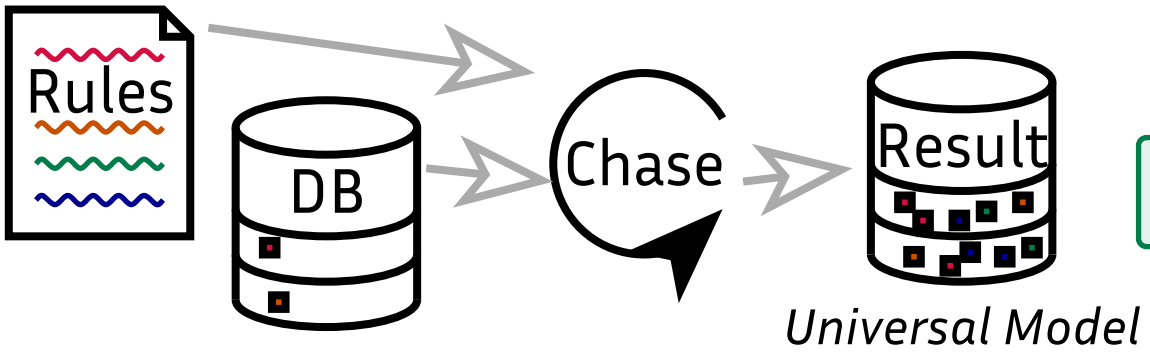

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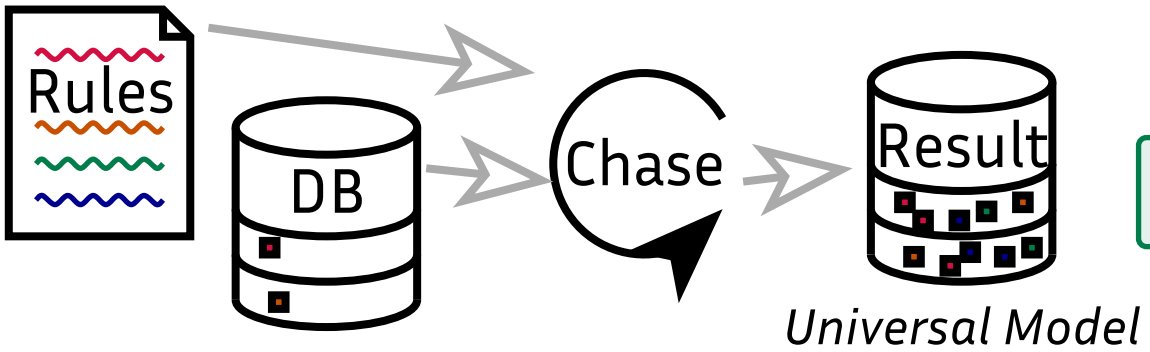

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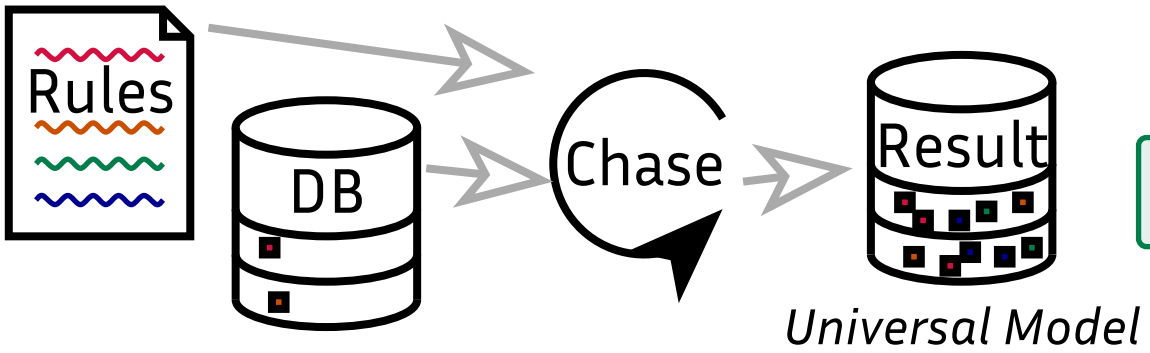

Universal Model

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Skolem Chase

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Step 2: $\mathrm{R}(f_z(d), f_z(f_z(d)))$, ...
Step N: ...

Restricted Chase

DB: $\mathrm{R}(c, d)$
Step 1: $\mathrm{R}(d, f_z(d)), \mathrm{R}(f_z(d), d)$

We still use Skolem functions instead of nulls (as a naming convention).

Formal Verification – Why Bother? 🤷

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Theory 🤓 – *Proving things is hard and intuitions might be deceiving.*

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Lean 🎉 – *Let's treat theorems and proofs like (open source) software.*

- A formal library can function as ground truth for both theory and practice.
- Mathlib and CSLib receive a lot of attention. Baanen et al. (2026), Barrett et al. (2026)

Lean Crash Course ⌨️

```
-- Let's (re)define the naturals ...
inductive Nat' where
| zero : Nat'
| succ : Nat' -> Nat'
```

Lean Crash Course ⌨️

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-- ... addition ...
def Nat'.add (n : Nat') : Nat' -> Nat'
| .zero => n
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Lean Crash Course ⌨

We show the proof state here.

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-- ... and prove that add is associative.
theorem Nat'.add_assoc (n m k : Nat') : n.add (m.add k) = (n.add m).add k := by
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n m k : Nat
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theorem Nat'.add_assoc (n m k : Nat') : n.add (m.add k) = (n.add m).add k := by
  induction k with
  | zero => rfl -- both sides are definitionally equal
```

```
n m : Nat
⊢ n.add (m.add 0) = (n.add m).add 0
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  | succ k ih => -- after unfolding the definition, applying the ih closes the goal
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n m k : Nat
ih : n.add (m.add k) = (n.add m).add k
⊢ n.add (m.add k.succ) = (n.add m).add k.succ
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    simp only [Nat'.add]
```

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-- ... and prove that add is associative.
theorem Nat'.add_assoc (n m k : Nat') : n.add (m.add k) = (n.add m).add k := by
  induction k <;> grind [add] -- or simply use grind in all cases
```

Goals accomplished 🎉

Cut to the Chase! 📏

DB: $\mathrm{R}(c,\ d)$ *(Skolem)*
Step 1: $\mathrm{R}(d,\ f_z(d)), \mathrm{R}(f_z(d),\ d)$
Step 2: $\mathrm{R}(f_z(d),\ f_z(f_z(d)))$, ...
Step N: ...

$$\mathrm{R}(x,\ y) \rightarrow \exists z.\ \mathrm{R}(y,\ z) \land \mathrm{R}(z,\ y)$$

DB: $\mathrm{R}(c,\ d)$ *(Restricted)*
Step 1: $\mathrm{R}(d,\ f_z(d)), \mathrm{R}(f_z(d),\ d)$

How do we capture the differences (Skolem/Restricted and Finite/Infinite)?

Cut to the Chase! 📏

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-- Skolem/Restricted behavior can be generalized to checking "obs"
structure Chase (obs : ObsolescenseCondition) (rs : RuleSet) where
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```
-- Skolem/Restricted behavior can be generalized to checking "obs"
structure Chase (obs : ObsolescenseCondition) (rs : RuleSet) where
  seq : List (ChaseNode obs rs) -- does not work...
  <conditions>
```

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-- Skolem/Restricted behavior can be generalized to checking "obs"
structure Chase (obs : ObsolescenseCondition) (rs : RuleSet) where
  seq : Nat -> (ChaseNode obs rs) -- still does not work...
  <conditions>
```

Cut to the Chase! 📏

DB: $\mathrm{R}(c,\, d)$ *(Skolem)*
Step 1: $\mathrm{R}(d,\, f_z(d)), \mathrm{R}(f_z(d),\, d)$
Step 2: $\mathrm{R}(f_z(d),\, f_z(f_z(d)))$, ...
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How do we capture the differences (Skolem/Restricted and Finite/Infinite)?

```
-- Skolem/Restricted behavior can be generalized to checking "obs"
structure Chase (obs : ObsolescenseCondition) (rs : RuleSet) where
  seq : Nat -> Option (ChaseNode obs rs) -- getting somewhere!
  <conditions>
```

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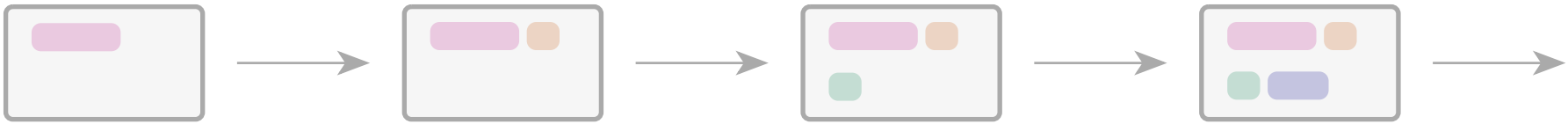
How do we capture the differences (Skolem/Restricted and Finite/Infinite)?

```
-- Skolem/Restricted behavior can be generalized to checking "obs"
structure Chase (obs : ObsolescenseCondition) (rs : RuleSet) where
  seq : PossiblyInfiniteList (ChaseNode obs rs)
  <conditions>
```

Basic ingredients of our framework 🍳

We need to be able to talk about the chase to state theorems in the first place.



1.
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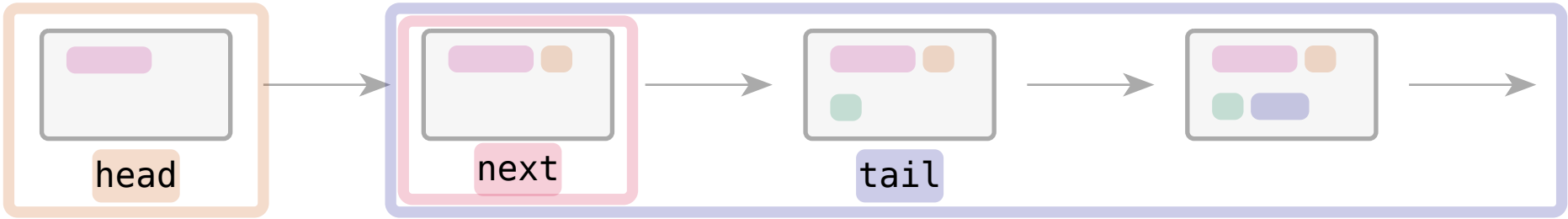

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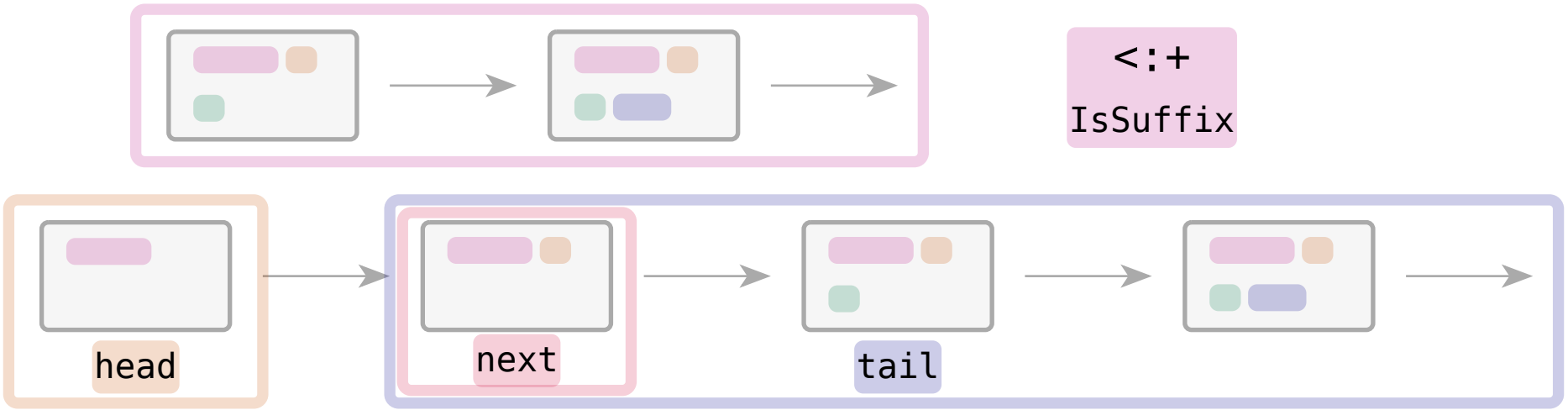

1. Basic (coinductive) accessors: `head`, `next` and `tail`
2. A suffix relation `<:+` on chase sequences.
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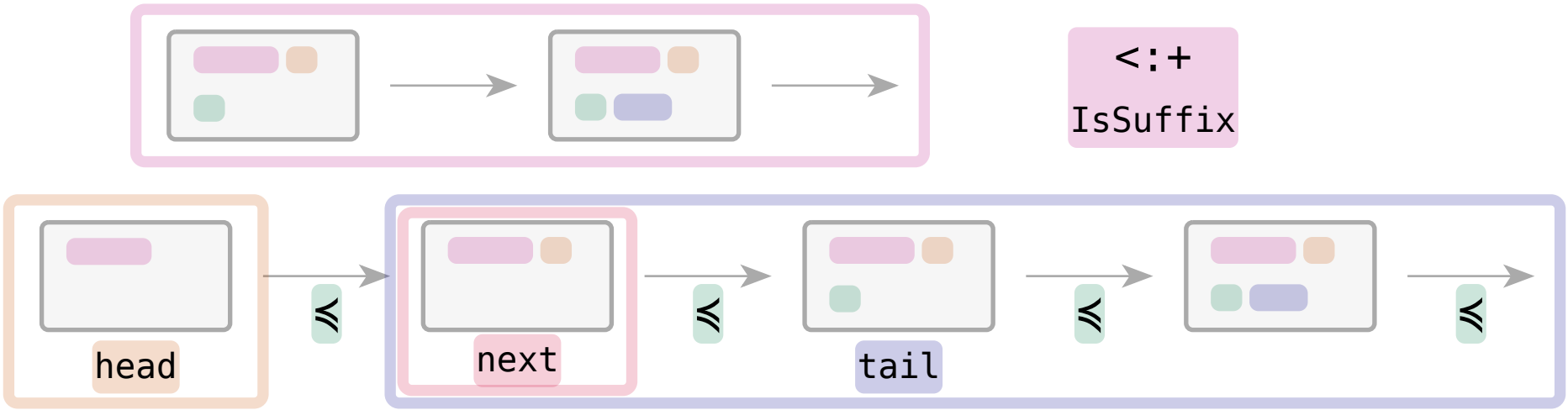

1. Basic (coinductive) accessors: `head`, `next` and `tail`
2. A suffix relation `<:+` on chase sequences.
3. A predecessor relation $\preccurlyeq$ on nodes in a single chase sequence.
4.

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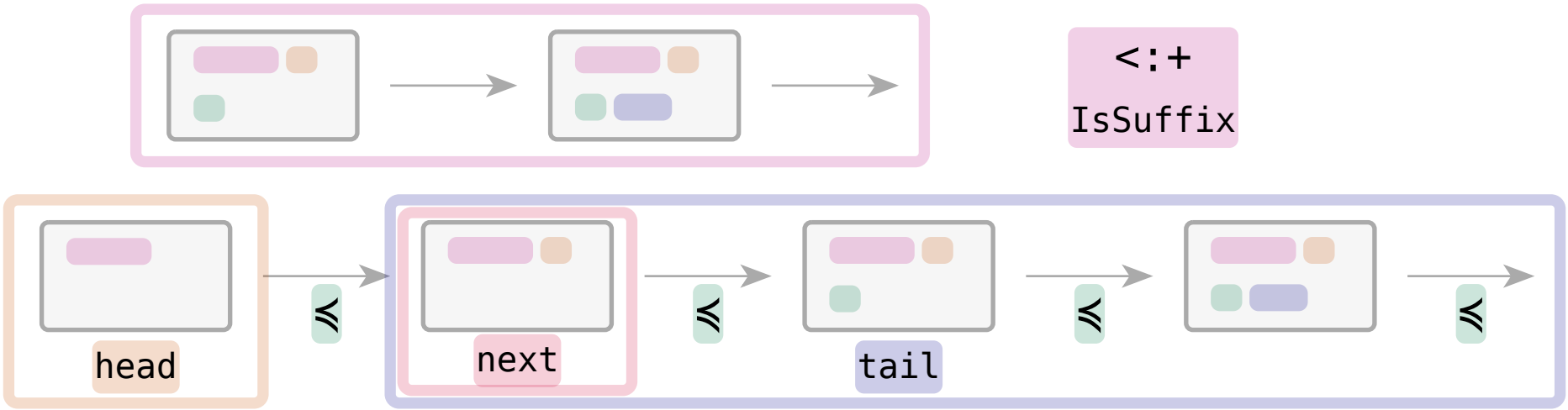

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3. A predecessor relation $\preccurlyeq$ on nodes in a single chase sequence.
4. An induction principle for nodes in the chase (next slide).

Defining our own induction principle 👀

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theorem facts_node_subset_every_mem :
    ∀ node ∈ cd, cd.head.facts ⊆ node.facts := by
  intro node node_mem
  let node' : cd.Node := ⟨node, node_mem⟩
  show cd.head.facts ⊆ node'.val.facts
  induction node' using mem_rec with
  | head => apply Set.subset_refl
  | step cd2 suffix subset next next_eq =>
    apply Set.subset_trans subset
    rw [cd2.facts_next next_eq]
    apply Set.subset_union_of_subset_left
    apply Set.subset_refl
```

```
theorem facts_node_subset_of_prec :
    node1 ≼ node2 ->
    node1.val.facts ⊆ node2.val.facts := by
  rintro ⟨cd2, suffix, head_eq, mem⟩
  rw [← head_eq]
  apply facts_node_subset_every_mem
  exact mem
```

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Results that are already part of the library 📋

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   - `¬cb.has_alt_match → cb.result.isStrongCore`
3. Each chase an every rule set that fulfills an MFA-like condition terminates; generalizing (Grau et al. 2013, Carral et al. 2017, Gerlach and Carral 2023a)
   - Termination conditions are generic over an `ObsolescenceCondition`.
   - ```
     rs.isMfa rs_finite (BlockingObsolescence obs rs) →
     rs.terminates obs (RegularChaseNode obs rs)
     ```
     ```

Results that are already part of the library 📋

1. The cha
   - Involv
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3. Each ch … es;
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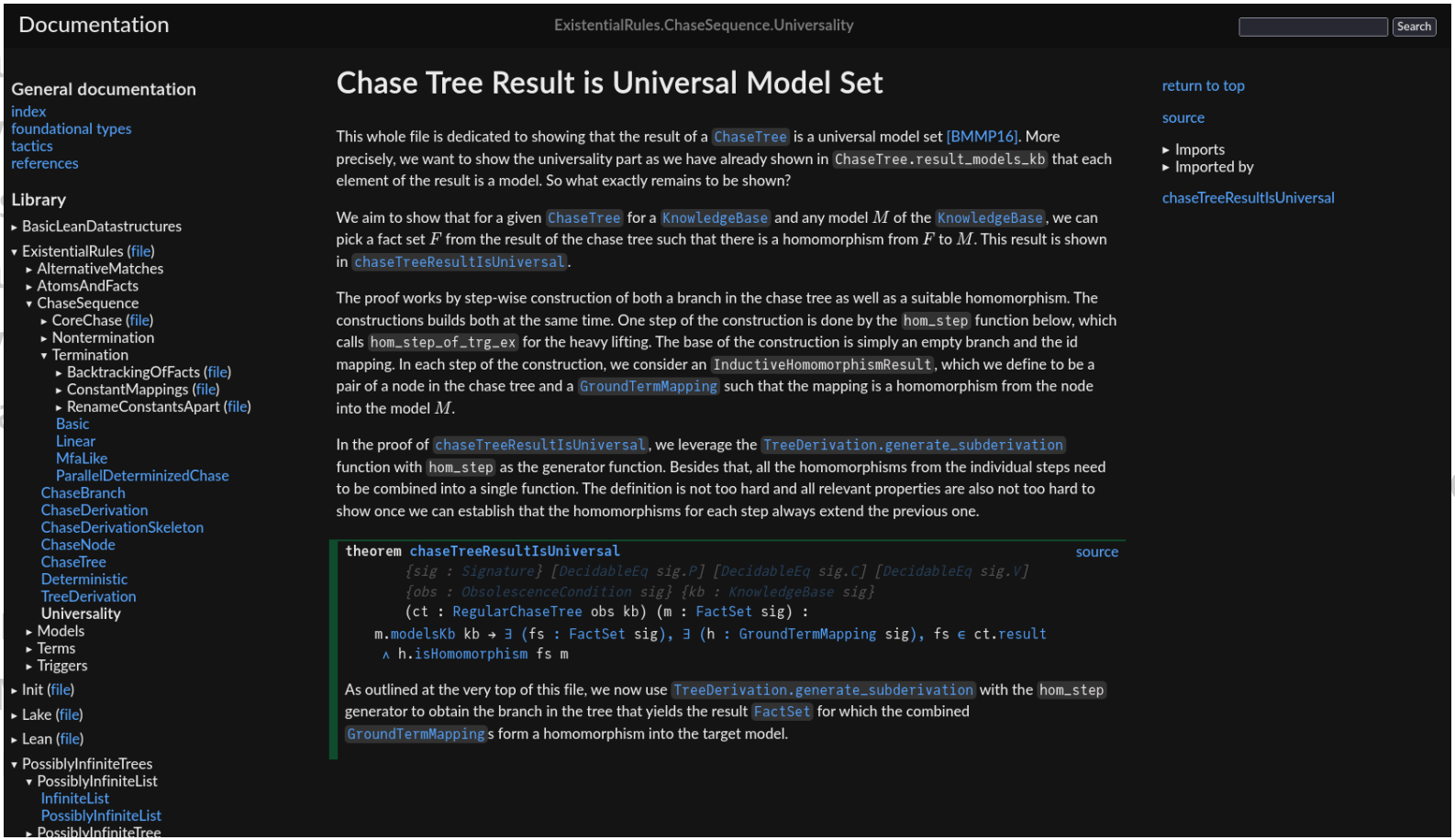
   - rs.isH
     rs.te



Lessons Learned & Outlook 🔭

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Invest in Frameworks 🚀 – *Think about what API you would expect as user.*

- Go bottom up and have sensible mini-libraries.
- This should involve (a lot of) obvious theorems (many of them provable via `rfl`).
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What's next? 🔮

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- An executable (naive) chase procedure.
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Visit https://dmfa.dev/lean 🤓

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