TECHNISCHE UNIVERSITÄT DRESDEN

DATABASE THEORY

Lecture 16: Path Queries

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Knowledge-Based Systems

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Review: Regular Path Queries

Idea: use regular expressions to navigate over paths

Let's consider a simplified graph model, where a graph is given by:

- Set of nodes N (without additional labels)
- Set of edges E, labelled by a function $\lambda : E \to L$, where L is a finite set of labels

Definition 16.1: A regular expression over a set of labels L is an expression of the following form:

$$E ::= L \mid (E \circ E) \mid (E + E) \mid E^*$$

A regular path query (RPQ) is an expression of the form $E(s, t)$, where E is a regular expression and s and t are terms (constants or variables).

Semantics of Regular Path Queries

As usual, a regular expression E matches a word $w = \ell_1 \cdots \ell_n$ if any of the following conditions is satisfied:

- $E \in L$ is a label and $w = E$.
- $E = (E_1 \circ E_2)$ and there is $i \in \{0, \ldots, n\}$ such that E_1 matches $\ell_1 \cdots \ell_i$ and E_2 matches $\ell_{i+1} \cdots \ell_n$ (the words matched by E_1 and E_2 can be empty if $i = 0$ or $i = n$, respectively).
- $E = (E_1 + E_2)$ and w is matched by E_1 or by E_2
- $E = E_1^*$ and w has the form $w_1 w_2 \cdots w_m$ for $m \geq 0$, where each word w_i is matched by E_1

Definition 16.2: Let a and b be constants and x and y be variables. An RPQ $E(a, b)$ is entailed by a graph G if there is a directed path from node a to node b that is labelled by a word matched by E. The answers to RPQs $E(x, y)$, $E(x, b)$, and $E(a, y)$ are defined in the obvious way.

Extending the Expressive Power of RPQs

Regular path queries can be used to express typical reachability queries, but are still quite limited $\leadsto$ extensions

2-Way Regular Path Queries (2RPQs)

- For every label $\ell \in L$, also introduce a converse label ℓ^-
- Allow converse labels in regular expressions
- Matched paths can follow edges forwards or backwards

Conjunctive Regular Path Queries (CRPQs)

- Extend conjunctive queries with RPQs
- RPQs can be used like binary query atoms
- Obvious semantics

Conjunctive 2-Way Regular Path Queries (C2RPQs) combine both extensions

C2RPQs: Examples

All ancestors of Alice:

$$((\text{father} + \text{mother}) \circ (\text{father} + \text{mother})^*)(\text{alice}, y)$$

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Pairs of stops connected by tram lines 3 and 8:

$$(\mathsf{nextStop3} \circ \mathsf{nextStop3}^*)(x, y) \land (\mathsf{nextStop8} \circ \mathsf{nextStop8}^*)(x, y)$$

Complexity of RPQs

A nondeterministic algorithm for Boolean RPQs:

- Transform regular expression into a finite automaton
- Starting from the first node, guess a matching path
- When moving along path, advance state of automaton
- Accept if the second node is reached in an accepting state
- Reject if path is longer than size of graph $\times$ size of automaton

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Space requirements when assuming query (and automaton) fixed: pointer to current node in graph, pointer to current state of automaton, counter for length of path
$\leadsto$ NL algorithm

Conversely, reachability in an unlabelled graph is hard for NL
$\leadsto$ RPQ matching is NL-complete (data complexity)

(Combined/query complexity is in P, as we will see below)

Complexity of C2RPQs

We already know:

- CQ matching is in AC^0 (data complexity) and NP-complete (query and combined complexity)
- RPQ matching is NL-complete (data) and in P (query/combined)
- $\text{AC}^0 \subset \text{NL}$ and $\text{NL} \subseteq \text{NP}$

$\leadsto$ C2RPQs are NP-hard (combined/query) and NL-hard (data)

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It's not hard to show that these bounds are tight:

Theorem 16.3: C2RPQ matching is NP-complete for combined and query complexity, and NL-complete for data complexity.

(C2)RPQs and Datalog

How do path queries relate to Datalog?

We already know:

- Datalog is ExpTime-complete (combined/query) and P-complete (data)
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Indeed, we can express regular expressions in Datalog

For simplicity, assume that we have a binary EDB predicate p_ℓ for each label $\ell \in L$
(other encodings would work just as well)

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If $E = E_1^*$ then

$$P_E = P_{E_1} \cup \{\mathsf{Q}_E(x, x) \leftarrow, \mathsf{Q}_E(x, z) \leftarrow \mathsf{Q}_E(x, y) \wedge \mathsf{Q}_{E_1}(y, z)\}$$

Reprise: Combined Complexity of 2RPQs

As a side effect, the previous translation shows that 2RPQs can be evaluated in P combined complexity:

- Each (2-way) regular expression E leads to a Datalog query $\langle Q_E, P_E \rangle$ of polynomial size
- Each rule in P_E has at most three variables
  $\leadsto$ the grounding of P_E for a graph with nodes N is of size $|P_E| \times |N|^3$
- propositional logic rules can be evaluated in polynomial time

$\leadsto$ polynomial time decision procedure

Expressing C2RPQs in Datalog

It is now easy to express C2RPQs in Datalog:

- Use the encoding of CQs in Datalog as shown in the exercise
- Express 2RPQ atoms in Datalog as just shown

Can every Datalog query over binary "labelled-edge" EDB predicates be expressed with (C2)RPQs?

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Can every Datalog query over binary "labelled-edge" EDB predicates be expressed with (C2)RPQs?

- This would imply P = NL (but not that NP = ExpTime!):
  unlikely but not known to be false
- However, there are stronger direct arguments that show the limits of C2RPQs (exercise)

Linear Datalog and Binary Datalog

Expressing 2RPQs in Datalog requires only restricted forms of Datalog:

Definition 16.4: A Datalog program is linear if each of its rules has at most one IDB atom in its body. A Datalog program is binary if all of its IDB predicates have arity at most two.

The following complexity results are known:

Theorem 16.5: Query answering in linear Datalog is NL-complete for data complexity, and PSpace-complete for combined and query complexity.
Combined complexity further drops to NP for binary Datalog.

$\leadsto$ complexity results that are more similar to (C2)RPQs . . .

2RPQs and Linear Datalog

The Datalog translation of 2RPQs does not lead to linear Datalog, but we can fix this.

We transform a regular expression E to a linear Datalog query $\langle \mathsf{Q}_E, P_E^{\mathsf{lin}} \rangle$:

- Construct a non-deterministic automaton $\mathcal{A}_E$ for E
- For every state q of $\mathcal{A}_E$, we use a binary IDB predicate S_q
- For the starting state q_0 of $\mathcal{A}_E$, we add a rule $\mathsf{S}_{q_0}(x, x) \leftarrow$
- For every transition $q \xrightarrow{\ell} q'$ of $\mathcal{A}_E$, we add a rule

$$\mathsf{S}_{q'}(x, z) \leftarrow \mathsf{S}_q(x, y) \wedge \mathsf{p}_\ell(y, z)$$

- For every final state q_f of $\mathcal{A}_E$, we add a rule

$$\mathsf{Q}_E(x, y) \leftarrow \mathsf{S}_{q_f}(x, y)$$

Two-way queries can be captured by allowing two-way transitions.

Linear Datalog vs. 2RPQs

So all 2RPQs can be expessed in linear Datalog
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Is the converse also true?

No. Counterexample:

$$\mathsf{Query}(x,z) \leftarrow \mathsf{p}_a(x,y) \wedge \mathsf{p}_b(y,z)$$
$$\mathsf{Query}(x,z) \leftarrow \mathsf{p}_a(x,x') \wedge \mathsf{Query}(x',z') \wedge \mathsf{p}_b(z',z)$$

The linear Datalog program matches paths with labels from $a^n b^n$
$\leadsto$ context-free, non-regular language
$\leadsto$ not expressible in (C2)RPQs

Intuition: linear Datalog generalises context-free languages

Query Optimisation for C2RPQs

Recall the basic static optimisation problems of database theory:

- Query containment
- Query equivalence
- Query emptiness

Which of these are decidable for (C2)RPQs?

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Observation: query emptiness is trivial

Containment for RPQs

Containment of Regular Path Queries corresponds to containment of regular expressions $\leadsto$ known to be decidable in PSpace

Proof sketch for checking $E_1 \sqsubseteq E_2$:

(1) Construct non-deterministic automata (NFAs), A_1 and A_2 for the regular expressions E_1 and E_2, respectively

(2) Construct an automaton $\bar{A}_2$ that accepts the complement of A_2.

(3) Construct the intersection $A_1 \cap \bar{A}_2$ of A_1 and $\bar{A}_2$

(4) Check if $A_1 \cap \bar{A}_2$ accepts a word (if yes, then there is a counterexample that disproves $E_1 \sqsubseteq E_2$; if no, then the containment holds)

Complexity estimate:
$A_1 \cap \bar{A}_2$ is exponential (blow-up by powerset construction in step (2)) but step (4) is possible by checking reachability on the state graph
$\leadsto$ NL algorithm on an exponential state graph
$\leadsto$ NPSpace algorithm (construct the state graph on the fly)
$\leadsto$ PSpace algorithm (Savitch's Theorem)

Containment for (C)2RPQs

Things are more tricky when adding converses and conjunctions

Theorem 16.6:

- Containment of 2RPQs is PSpace-complete
- Containment of C2RPQs is ExpSpace-complete

The proofs are more involved.

Automata-theoretic constructions are used, but with more complicated automata models and for somewhat different languages (there is no good “language of possible C2RPQ matches on a graph” $\rightsquigarrow$ consider language of possible proofs instead)

Query Optimisation for Path Queries

Decidable in PSpace (2RPQs) and ExpSpace (C2RPQs)

Should be compared to linear Datalog:

Theorem 16.7: Query containment for linear Datalog queries is undecidable.

Proof: see Lecture 13 (Post Correspondence Problem in Datalog – in fact, in linear Datalog) □

Query containment of (C2)RPQs is seeing essentially no adoption in practice
$\leadsto$ maybe the complexities are too high . . .
$\leadsto$ or maybe path query optimisers are just too primitive . . .
$\leadsto$ or maybe (current) real-world queries do not look as if they would benefit from this effort

Path Queries: Final Remarks on Expressivity

We have seen that C2RPQs are NL-complete for data
$\rightsquigarrow$ can all NL-complete queries be captured by a C2RPQ?

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No. For many reasons.

- C2RPQs have no disjunction ($\rightsquigarrow$ Unions of C2RPQs)
- C2RPQs have no negation

FO-queries with a binary transitive closure operator capture NL

Several (regular) extensions of path queries:

- Nested unary 2RPQs in regular expressions (“test operators”)
- Nested binary C2RPQs in regular expressions
- Other more expressive fragments of “regular Datalog”, e.g., Monadically Defined Queries

Summary and Outlook

Graph databases as an important class of “noSQL” databases

Two main data models

- Resource Description Framework (RDF)
- Property Graph

Path queries as common foundation of all graph query languages

- higher data complexities than CQs/FO queries
- lower complexities than Datalog queries
- decidable query optimisation

Next topics:

- Logical dependencies
- Query answering under constraints