



PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 5 Tabu Search

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Agenda

- 1 Introduction
- 2 Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
- 3 Local Search, Stochastic Hill Climbing, Simulated Annealing
- 4 **Tabu Search**
- 5 Answer-set Programming (ASP)
- 6 Constraint Satisfaction (CSP)
- 7 Structural Decomposition Techniques (Tree/Hypertree Decompositions)
- 8 Evolutionary Algorithms/ Genetic Algorithms

Tabu Search

Main Idea

- A **memory** forces the search to explore new areas of the search space
- Memorize solutions that have been **examined recently**. They become **tabu** points in next steps
- Tabu search is **deterministic**

Tabu Search and SAT

- SAT problem with $n = 8$ variables
- Initial (random) assignment $\mathbf{x} = (0, 1, 1, 1, 0, 0, 0, 1)$
- Evaluation function: **weighted sum** of number of satisfied clauses. Weights depend on the number of variables in the clause
- **Maximize** evaluation function (i.e. we're trying to satisfy all clauses)
- Random assignment provides $eval(\mathbf{x}) = 27$
- **Neighborhood** of \mathbf{x} consists of 8 solutions. **Evaluate** them and **select best**
- **At this stage, it is the same as hill-climbing**
- Suppose flipping 3rd variable generates best evaluation ($eval(\mathbf{x}') = 31$)
- **Memory** keeps track of actions

Recency-based Memory

- **Index** of flipped variable + **time** when it was flipped
- **Differentiate between older and more recent flips**
- **SAT: time stamp** for each position of solution vector M (initialized to 0)
- Value of time stamp provides information on **recency** of flip at position

Memory Vector

$M(i) = j$ (when $j \neq 0$)
 j is most recent iteration when i -th bit was flipped

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Assume information is **stored for at most 5 iterations**.

Alternative Interpretation

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 i -th bit was flipped $5 - j$ iterations ago

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Example

0	0	5	0	0	0	0	0
---	---	---	---	---	---	---	---

Memory after one iteration. 3rd bit is **tabu** for next 5 iterations.

Different Interpretations

1st Variant

- Stores iteration number of most recent flip
- Requires a current iteration counter t which is compared with memory values
- If $t - M(i) > 5$ forget
- Only requires updating a single entry, and increase the counter
- **Used in most implementations**

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2nd Variant

- Values are interpreted as number of iterations for which a position is **not available**
- **All** nonzero entries are decreased by one **at every iteration**

Example ctd.

- Initial assignment $\mathbf{x} = (0, 1, 1, 1, 0, 0, 0, 1)$
- After 4 additional iterations M :

3	0	1	5	0	4	2	0
---	---	---	---	---	---	---	---

- Most recent flip $M(4) = 5$
- **Current solution:** $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$

Example ctd.

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Neighborhood of \mathbf{x}

$$\mathbf{x}_1 = (0, 1, 0, 0, 0, 1, 1, 1)$$

$$\mathbf{x}_2 = (1, 0, 0, 0, 0, 1, 1, 1)$$

$$\mathbf{x}_3 = (1, 1, 1, 0, 0, 1, 1, 1)$$

$$\mathbf{x}_4 = (1, 1, 0, 1, 0, 1, 1, 1)$$

$$\mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1, 1)$$

$$\mathbf{x}_6 = (1, 1, 0, 0, 0, 0, 1, 1)$$

$$\mathbf{x}_7 = (1, 1, 0, 0, 0, 1, 0, 1)$$

$$\mathbf{x}_8 = (1, 1, 0, 0, 0, 1, 1, 0)$$

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$$\mathbf{x}_6 = (1, 1, 0, 0, 0, 0, 1, 1)$$

$$\mathbf{x}_7 = (1, 1, 0, 0, 0, 1, 0, 1)$$

$$\mathbf{x}_8 = (1, 1, 0, 0, 0, 1, 1, 0)$$

TABU, best evaluation $eval(\mathbf{x}_5) = 32$, **decrease!**

Example ctd.

- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$
- New solution: $\mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1, 1)$ with $eval(\mathbf{x}_5) = 32$

3	0	1	5	0	4	2	0
---	---	---	---	---	---	---	---

changes to:

2	0	0	4	5	3	1	0
---	---	---	---	---	---	---	---

Example ctd.

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Policy might be too restrictive

- What if tabu neighbor \mathbf{x}_6 provides **excellent evaluation score**?
- Make search more flexible: **override** tabu classification if solution is **outstanding**

⇒ **aspiration criterion**

Long-term Memory

Question

- 1 What is stored in **long-term memory** (think of SAT as an example)?
- 2 How can we **escape local optima** with help of a long-term memory?



Frequency-based Memory

- Operates over a longer horizon
- SAT: vector H serves as long-term memory.
 - Initialized to 0, at any stage of the search

$$H(i) = j$$

interpreted as: during last h (horizon) iterations, the i -th bit was flipped j times

- Usually horizon is large
- After 100 iterations with $h = 50$, long-term memory H might have the following values

5	7	11	3	9	8	1	6
---	---	----	---	---	---	---	---

- Shows **distribution** of moves throughout the last 50 iterations

Diversity of Search

Frequency-based memory provides information about which flips have been **under-represented** or not represented.

⇒ we can **diversify** the search by **exploring these possibilities**

Use of Long-term Memory

Special Circumstances

- Situations where **all non-tabu moves** lead to **worse solution**
- To make a meaningful decision about which direction to explore next
- Typically: **most frequent** moves are **less attractive**
- Value of evaluation score is decreased by some **penalty measure** that depends on frequency, final score implies the winner

Example SAT

- Assume value of current solution is $eval(\mathbf{x}) = 35$
- Non-tabu flips 2, 3 and 7 have values 30, 33, 31
- None of tabu moves provides value greater than 37 (highest value so far)
⇒ we can't apply **aspiration criterion**

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⇒ we can't apply **aspiration criterion**
- Frequency based-memory and evaluation function for new solution \mathbf{x}' is

$$eval(\mathbf{x}') - penalty(\mathbf{x}')$$

- $penalty(\mathbf{x}') = 0.7 \times H(i)$, where 0.7 coefficient, $H(i)$ value from long-term memory H :

7	for solution created by flipping 2nd bit
11	for solution created by flipping 3rd bit
1	for solution created by flipping 7nd bit

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- New scores are:

$30 - 0.7 \times 7 = 25.1$	2nd bit
$33 - 0.7 \times 11 = 25.3$	3nd bit
$31 - 0.7 \times 1 = 30.3$	7th bit

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Diversify Search

Including frequency values in a penalty measure for evaluating solutions.

Further Options to Diversify Search

We might add additional rules:

- **Aspiration by default:** select the **oldest** of all considered
- **Aspiration by search direction:** **memorize** whether or not the performed moves generated any **improvement**
- **Aspiration by influence:** measures the degree of change of the new solution
 - a) in terms of the **distance** between old and new solution
 - b) change in **solution's feasibility**, if we deal with a constraint problem
 - **Intuition:** particular move has a **larger influence** if a **larger step** was made from old to new solution

Tabu Search and the TSP

- **Move:** swap two cities in a particular solution
- Current solution: (2, 4, 7, 5, 1, 8, 3, 6)
- 28 neighbors $\binom{8}{2} = \frac{7 \cdot 8}{2} = 28$
- **Recency-based memory:** swap of cities i and j in i -th row and j -th column (for $i < j$)
- Maintain number of remaining iterations for which swap stays on tabu list
- **Frequency-based memory:** same structure; indicate totals of all swaps within horizon $h = 50$

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	2	3	4	5	6	7	8	
1								1
2	■							2
3		■						3
4			■					4
5				■				5
6					■			6
7						■		7

Tabu Search and the TSP ctd.

- Assume both memories initialized to zero and 500 iterations have been completed
- Current solution: (7, 3, 5, 6, 1, 2, 4, 8) with length: 173, best solution so far 171

	2	3	4	5	6	7	8	
1	0	0	1	0	0	0	0	1
2	■	0	0	0	5	0	0	2
3		■	0	0	0	4	0	3
4			■	3	0	0	0	4
5				■	0	0	2	5
6					■	0	0	6
7						■	0	7

	2	3	4	5	6	7	8	
1	0	2	3	3	0	1	1	1
2	■	2	1	3	1	1	0	2
3		■	2	3	3	4	0	3
4			■	1	1	2	1	4
5				■	4	2	1	5
6					■	3	1	6
7						■	6	7

left: recency-based memory; right: frequency-based memory

Summary

- Simulated annealing and tabu search are both **design to escape local optima**
- Tabu search makes **uphill moves only** when it is **stuck in local optima**
- Simulated annealing can make uphill moves at any time
- Simulated annealing is **stochastic**, tabu search is **deterministic**
- Compared to classic algorithms, both work on **complete solutions**. One can halt them at any iteration and obtain a possible solution
- Both have **many parameters** to worry about

References



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