Exercise 12: Dependencies

Database Theory

2023-07-04

Maximilian Marx, Markus Krötzsch

Exercise 1

Exercise. Let $\mathcal{L}$ be a fragment of first-order logic for which finite model entailment and arbitrary model entailment coincide, i.e., for every $\mathcal{L}$-theory $\mathcal{T}$ and every $\mathcal{L}$-formula φ, we find that φ is true in all models of $\mathcal{T}$ if and only if φ is true in all finite models of $\mathcal{T}$.

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   - One of these two procedures will terminate; run them in parallel.

Exercise 2

Consider the following set of tgds Σ:

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\begin{aligned}
\mathsf{A}(x) &\rightarrow \exists y.\, \mathsf{R}(x,y) \wedge \mathsf{B}(y) \\
\mathsf{B}(x) &\rightarrow \exists y.\, \mathsf{S}(x,y) \wedge \mathsf{A}(y) \\
\mathsf{R}(x,y) &\rightarrow \mathsf{S}(y,x) \\
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- No, the oblivious chase does not universally terminate for Σ. In particular, it does not terminate on the critical instance $\mathcal{I}_\star$.

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Solution.

- No, the oblivious chase does not universally terminate for Σ. In particular, it does not terminate on the critical instance $\mathcal{I}_\star$.
- No, the restricted chase does not, in general, universally terminate for Σ either.
- However, if the full dependencies are prioritised in the restricted chase, then the chase terminates on all database instances.

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$$\mathsf{B}(x) \rightarrow \exists y.\, \mathsf{S}(x, y) \wedge \mathsf{A}(x)$$
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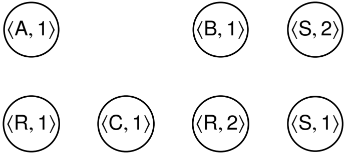

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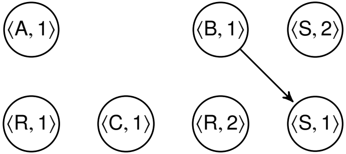

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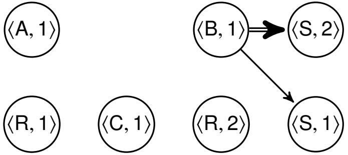

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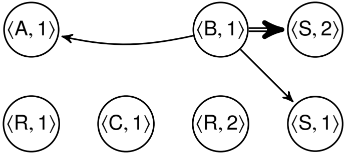

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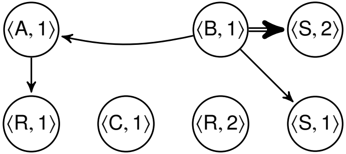

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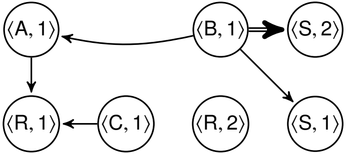

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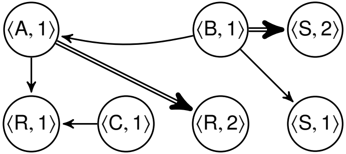

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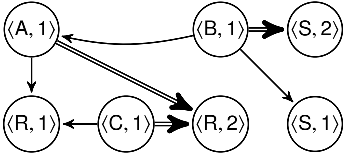

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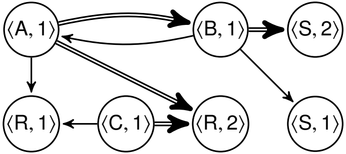

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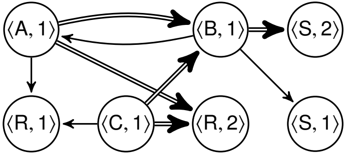
- If x is universally quantified and occurs at position $\langle q, j\rangle$ in the body of ρ, then there is an edge $\langle q, j\rangle \to \langle p, i\rangle$.
- If x is existentially quantified and another variable y occurs at position $\langle q, j\rangle$ in the body of ρ, then there is a special edge $\langle q, j\rangle \Rightarrow \langle p, i\rangle$.

Σ is *weakly acyclic* if its dependency graph does not contain a cycle that involves a special edge.



Exercise 3

Exercise. Is the following set of tgds Σ weakly acyclic?

$$B(x) \rightarrow \exists y.\ S(x,y) \wedge A(x)$$
$$A(x) \wedge C(x) \rightarrow \exists y.\ R(x,y) \wedge B(y)$$

Does the skolem chase universally terminate for Σ?
Solution.

Definition (Weak Acyclicity, Lecture 18, slide 19)

A *predicate position* is a pair $\langle p, i\rangle$ with p a predicate symbol and $1 \leq i \leq \text{arity}(p)$. For an atom $p(t_1, \ldots, t_n)$, the *term at position* $\langle p, i\rangle$ is t_i. The *dependency graph* of a tgd set Σ has the set of all positions in predicates of Σ as its nodes. For every rule ρ, and every variable x at position $\langle p, i\rangle$ in the head of ρ, the graph contains the following edges:

- If x is universally quantified and occurs at position $\langle q, j\rangle$ in the body of ρ, then there is an edge $\langle q, j\rangle \rightarrow \langle p, i\rangle$.
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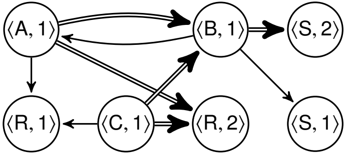
Σ is *weakly acyclic* if its dependency graph does not contain a cycle that involves a special edge.



1. Since $\langle A, 1\rangle \Longrightarrow \langle B, 1\rangle \longrightarrow \langle A, 1\rangle$ is a cycle involving a special edge, Σ is not weakly acyclic.

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Exercise. Is the following set of tgds Σ weakly acyclic?

$$\mathsf{B}(x) \to \exists y.\, \mathsf{S}(x,y) \wedge \mathsf{A}(x)$$
$$\mathsf{A}(x) \wedge \mathsf{C}(x) \to \exists y.\, \mathsf{R}(x,y) \wedge \mathsf{B}(y)$$

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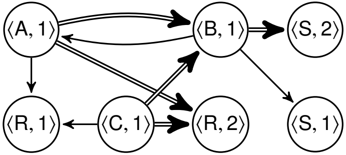

1. Since $\langle A, 1\rangle \Longrightarrow \langle B, 1\rangle \longrightarrow \langle A, 1\rangle$ is a cycle involving a special edge, Σ is not weakly acyclic.
2. The skolem chase for Σ terminates on the critical instance $\mathcal{I}_\star$, therefore it terminates universally.

Exercise 4

Termination of the oblivious (resp. restricted) chase over a set of tgds Σ implies the existence of a finite universal model for Σ. Is the converse true? That is, does the existence of a finite universal model for Σ imply termination of the oblivious (resp. restricted) chase?

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Solution.

- No, consider, e.g., $\Sigma = \{\, \mathrm{A}(x) \rightarrow \exists y.\, \mathrm{R}(x, y) \wedge A(y), \rightarrow \exists x.\, A(x) \,\}$.

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- $\mathcal{M} = \{\, \mathrm{A}(n), \mathrm{R}(n, n) \,\}$ with n a null is a finite universal model of Σ.

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Solution.

- No, consider, e.g., $\Sigma = \{ \mathrm{A}(x) \rightarrow \exists y.\, \mathrm{R}(x, y) \wedge A(y), \rightarrow \exists x.\, A(x) \}$.
- $\mathcal{M} = \{ \mathrm{A}(n), \mathrm{R}(n, n) \}$ with n a null is a finite universal model of Σ.
- However, neither the oblivious nor the restricted chase for Σ terminates on the empty database instance.

Exercise 5

Consider a set of tgds Σ that does not contain any constants. A term is *cyclic* if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol f syntactically occurs in t_i. Then Σ is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup \mathcal{I}_\star$, where $\mathcal{I}_\star$ is the critical instance.

Show the following claims:

1. Checking MFA membership is decidable.
2. Is the set of tgds from Exercise 12.3 MFA?

$$\mathsf{B}(x) \rightarrow \exists y.\, \mathsf{S}(x, y) \wedge \mathsf{A}(x)$$
$$\mathsf{A}(x) \wedge \mathsf{C}(x) \rightarrow \exists y.\, \mathsf{R}(x, y) \wedge \mathsf{B}(y)$$

3. If a set of tgds Σ without constants is MFA, then the skolem chase universally terminates for Σ.

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Solution.

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3. If a set of tgds Σ without constants is MFA, then the skolem chase universally terminates for Σ.

Solution.

1. ▶ For a fixed set Σ of tgds, the number of non-cyclic terms over the signature is (double-exponentially) bounded; call this bound ℓ.

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Consider a set of tgds Σ that does not contain any constants. A term is *cyclic* if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol f syntactically occurs in t_i. Then Σ is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup \mathcal{I}_\star$, where $\mathcal{I}_\star$ is the critical instance.
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Solution.

1. - For a fixed set Σ of tgds, the number of non-cyclic terms over the signature is (double-exponentially) bounded; call this bound ℓ.
   - We can therefore decide MFA membership by computing the chase:

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Show the following claims:

1. Checking MFA membership is decidable.
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$$\mathsf{B}(x) \to \exists y.\ \mathsf{S}(x, y) \land \mathsf{A}(x)$$
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1. - For a fixed set Σ of tgds, the number of non-cyclic terms over the signature is (double-exponentially) bounded; call this bound ℓ.
   - We can therefore decide MFA membership by computing the chase:
   - If, at any point, a cyclic term appears, we know that Σ is not MFA.

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2.
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1.
   - For a fixed set Σ of tgds, the number of non-cyclic terms over the signature is (double-exponentially) bounded; call this bound ℓ.
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   - No cyclic terms, so Σ is MFA.

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Consider a set of tgds Σ that does not contain any constants. A term is *cyclic* if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol f syntactically occurs in t_i. Then Σ is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup \mathcal{I}_\star$, where $\mathcal{I}_\star$ is the critical instance.
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3. If a set of tgds Σ without constants is MFA, then the skolem chase universally terminates for Σ.

Solution.

1.
   - For a fixed set Σ of tgds, the number of non-cyclic terms over the signature is (double-exponentially) bounded; call this bound ℓ.
   - We can therefore decide MFA membership by computing the chase:
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2.
   - Critical instance $\mathcal{I}_\star = \{\mathsf{A}(\star), \mathsf{B}(\star), \mathsf{C}(\star), \mathsf{S}(\star, \star), \mathsf{R}(\star, \star)\}$.
   - Chase for $\Sigma \cup \mathcal{I}_\star$: $\{\mathsf{S}(\star, f(\star)), \mathsf{R}(\star, g(\star)), \mathsf{B}(g(\star)), \mathsf{S}(g(\star), f(g(\star))), \mathsf{A}(g(\star))\} \cup \mathcal{I}_\star$
   - No cyclic terms, so Σ is MFA.
3.
   - Since Σ is MFA, the skolem chase for Σ on $\mathcal{I}_\star$ contains no cyclic terms.

Exercise 5

Consider a set of tgds Σ that does not contain any constants. A term is *cyclic* if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol f syntactically occurs in t_i. Then Σ is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup \mathcal{I}_\star$, where $\mathcal{I}_\star$ is the critical instance.
Show the following claims:

1. Checking MFA membership is decidable.
2. Is the set of tgds from Exercise 12.3 MFA?

$$\mathsf{B}(x) \rightarrow \exists y.\, \mathsf{S}(x, y) \wedge \mathsf{A}(x)$$
$$\mathsf{A}(x) \wedge \mathsf{C}(x) \rightarrow \exists y.\, \mathsf{R}(x, y) \wedge \mathsf{B}(y)$$

3. If a set of tgds Σ without constants is MFA, then the skolem chase universally terminates for Σ.

Solution.

1.
   - For a fixed set Σ of tgds, the number of non-cyclic terms over the signature is (double-exponentially) bounded; call this bound ℓ.
   - We can therefore decide MFA membership by computing the chase:
   - If, at any point, a cyclic term appears, we know that Σ is not MFA.
   - If the chase terminates without generating a cyclic term, we know that Σ is MFA.
   - If the chase does not terminate within ℓ steps, it must generate a cyclic term at some point, thus Σ is not MFA.
2.
   - Critical instance $\mathcal{I}_\star = \{\, \mathsf{A}(\star), \mathsf{B}(\star), \mathsf{C}(\star), \mathsf{S}(\star, \star), \mathsf{R}(\star, \star) \,\}$.
   - Chase for $\Sigma \cup \mathcal{I}_\star$: $\{\, \mathsf{S}(\star, f(\star)), \mathsf{R}(\star, g(\star)), \mathsf{B}(g(\star)), \mathsf{S}(g(\star), f(g(\star))), \mathsf{A}(g(\star)) \,\} \cup \mathcal{I}_\star$
   - No cyclic terms, so Σ is MFA.
3.
   - Since Σ is MFA, the skolem chase for Σ on $\mathcal{I}_\star$ contains no cyclic terms.
   - Since the number of non-cyclic terms is bounded, the chase must be finite.

Exercise 5

Consider a set of tgds Σ that does not contain any constants. A term is *cyclic* if it is of the form $f(t_1, \ldots, t_n)$ and, for some $i \in \{1, \ldots, n\}$, the function symbol f syntactically occurs in t_i. Then Σ is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of $\Sigma \cup \mathcal{I}_\star$, where $\mathcal{I}_\star$ is the critical instance.
Show the following claims:

1. Checking MFA membership is decidable.
2. Is the set of tgds from Exercise 12.3 MFA?

$$\mathsf{B}(x) \to \exists y.\, \mathsf{S}(x, y) \wedge \mathsf{A}(x)$$
$$\mathsf{A}(x) \wedge \mathsf{C}(x) \to \exists y.\, \mathsf{R}(x, y) \wedge \mathsf{B}(y)$$

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   - Critical instance $\mathcal{I}_\star = \{\, \mathsf{A}(\star), \mathsf{B}(\star), \mathsf{C}(\star), \mathsf{S}(\star, \star), \mathsf{R}(\star, \star) \,\}$.
   - Chase for $\Sigma \cup \mathcal{I}_\star$: $\{\, \mathsf{S}(\star, f(\star)), \mathsf{R}(\star, g(\star)), \mathsf{B}(g(\star)), \mathsf{S}(g(\star), f(g(\star))), \mathsf{A}(g(\star)) \,\} \cup \mathcal{I}_\star$
   - No cyclic terms, so Σ is MFA.
3.
   - Since Σ is MFA, the skolem chase for Σ on $\mathcal{I}_\star$ contains no cyclic terms.
   - Since the number of non-cyclic terms is bounded, the chase must be finite.
   - The skolem chase for Σ on the critical instance terminates, therefore the skolem chase for Σ is universally terminating.