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Winter Term 2025/2026

Complexity Theory

Exercise 4: NP-Completeness

10 November 2025

Exercise 4.1. Show that the following problem is NP-complete:

Input:	A propositional formula φ in CNF
Question:	Does φ have at least 2 different satisfying assignments?

Exercise 4.2. We recall some definitions.

- Given some language **L**, $\mathbf{L} \in \mathrm{coNP}$ if, and only if, $\overline{\mathbf{L}} \in \mathrm{NP}$.
- **L** is coNP-hard if, and only if, $\mathbf{L}' \leq_p \mathbf{L}$ for every $\mathbf{L}' \in \mathrm{coNP}$.
- **L** is coNP-complete if, and only if, $\mathbf{L} \in \mathrm{coNP}$ and **L** is coNP-hard.

Show that if any coNP-complete problem is in NP, then $\mathrm{NP} = \mathrm{coNP}$.

Exercise 4.3. If G is an undirected graph, a *vertex cover* of G is a subset of the nodes where every edge of G touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size.

$$\textbf{Vertex-Cover} = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover.}\}$$

Show that **Vertex-Cover** is NP-complete.

Hint:
Try to find a reduction from 3-Sat

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Exercise 4.4. Show that if a language L is NP-complete, then $\overline{L}$ is coNP-complete.

Exercise 4.5. Show that finding paths of a given length in undirected graphs, i.e.,

$$\textbf{Path} = \{\ \langle G, s, t, k\rangle \mid G \text{ contains a simple path from } s \text{ to } t \text{ of length } k\ \}$$

is NP-complete.

∗ **Exercise 4.6.** Let $A \subseteq 1^*$. Show that if A is NP-complete, then P = NP.

Proceed as follows: Consider a polynomial-time reduction f from SAT to A. For a formula φ, let φ_{0100} be the reduced formula where variables x_1, x_2, x_3, x_4 in φ are set to the values 0, 1, 0, 0, respectively. (The particular choice of 4 variables as well as of 0100 is arbitrary here) What happens when one applies f to all of these exponentially many reduced formulas?