Possibly Infinite Trees of Finite Degree

Working towards König's Lemma

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13.03.2025

Background / Short Motivation

My work revolves around the chase algorithm on existential rules.

(Database Theory and Knowledge Representation)

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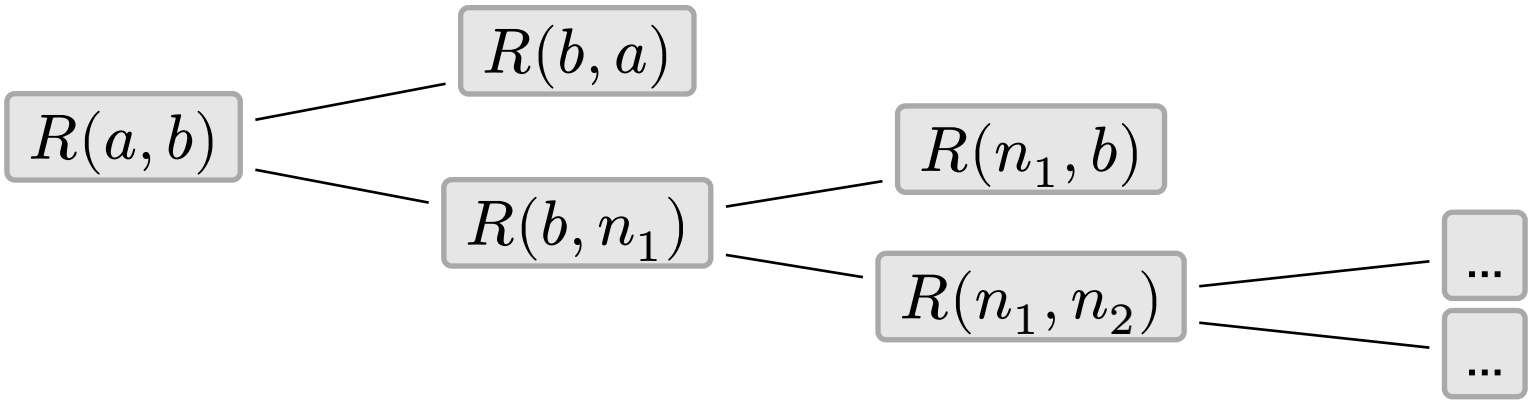

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We want the greatest fixed point instead, i.e. a coinductive definition!

It's time for some code!

1.
2.
3.
4.
5.

https://dmfa.dev/leaning-in-2025

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König's Lemma: (Special Case)
If each branch of a tree with finite degree is finite,
then there are only finitely many branches.

Possibly Infinite Tree - The Idea

```
def PossiblyInfiniteTree (a : Type u) := List Nat -> Option a
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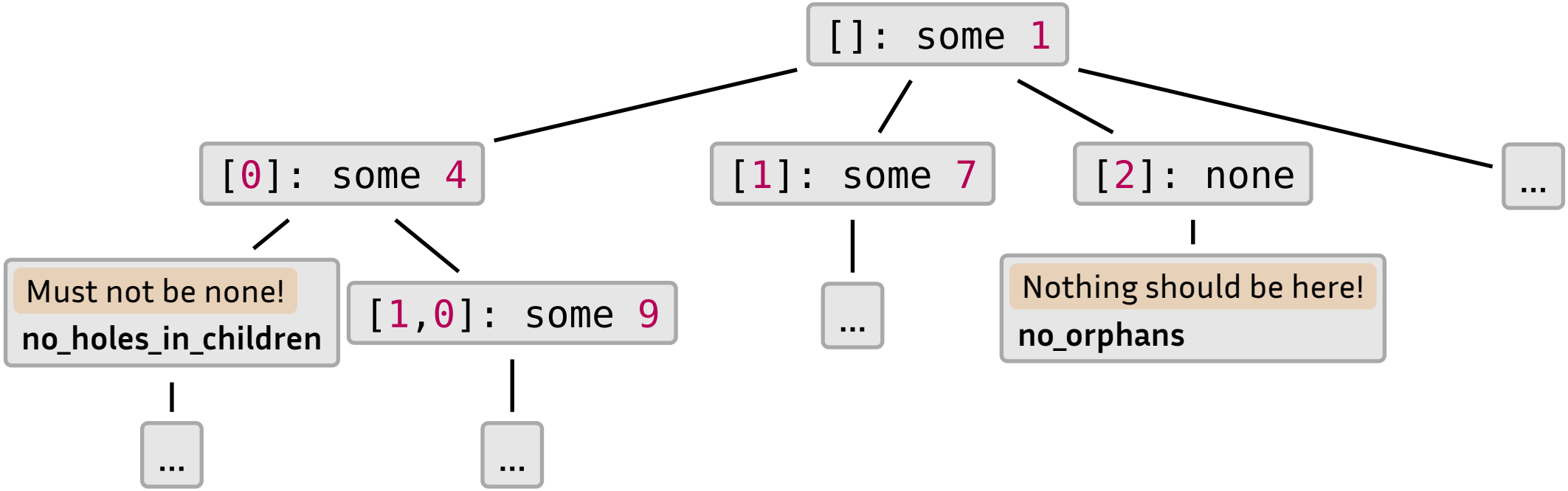

That's all!

Thank you so much for having me! 🤓

I hope you can get all your

`goals accomplished` 🎉

Feel free to reach out in Lean Zulipchat or via mail:

`lukas.gerlach@tu-dresden.de`

`hi@monsterkrampe.dev`