TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

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# Algorithmic Game Theory

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## Exercises 12

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**Problem 1.**
Consider the 3-person cooperative game with $v(C) = 2$ if $|C| \geq 2$ and $v(C) = 0$ otherwise. Show that $\{(1,1,0),(1,0,1),(0,1,1)\}$ is a stable set of this game.

**Problem 2.**
Show that the game from Problem 1 has (at least) one other stable set.

Hint: Consider the stable set given for "Hospitals and X-Ray Machines" in the lecture.

**Problem 3.**
Consider the following cooperative game: $P = \{1,2,3\}$ and

$$\begin{aligned} v(\{1\}) = 2, v(\{2\}) = 4, v(\{3\}) = 6 \\ v(\{1,2\}) = 6, v(\{1,3\}) = 8, v(\{2,3\}) = 12 \\ v(\{1,2,3\}) = 14 \end{aligned} \tag{1}$$

Do the following:

(a) Compute each player's marginal contributions to coalitions.

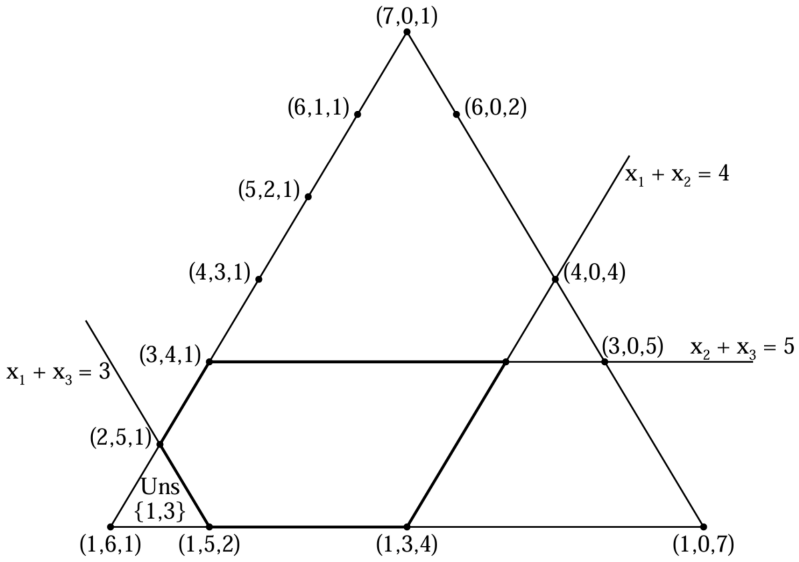
(b) Check whether there exists a dummy player.

**Problem 4.**
Consider the following cooperative game: $P = \{1,2,3\}$ and

$$\begin{aligned} v(\emptyset) = 0 \\ v(\{1\}) = 1, v(\{2\}) = 0, v(\{3\}) = 1 \\ v(\{1,2\}) = 4, v(\{1,3\}) = 3, v(\{2,3\}) = 5 \\ v(\{1,2,3\}) = 8 \end{aligned} \tag{2}$$

Note that the imputations are the points $(x_1, x_2, x_3)$ such that $x_1 + x_2 + x_3 = 8$ and $x_1 \geq 1, x_2 \geq 0, x_3 \geq 1$.

The set of all possible imputations is illustrated in the following triangle where we denote those imputations as *unstable* (abbreviated as Uns in the picture) that are not coalitionally rational.




Answer the following questions:

(a) Which of the imputations are not coalitionally rational? A first set of such unstable imputations is already given through the coalition $\{1,3\}$.

(b) Identify the core of the game based on the unstable imputations.

(c) Is the vector of Shapley values contained in the core? For this exercise, compute the Shapley values based on the original formula.