Exercise 1

SAT Solving

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Exercise 1.1 (Sub-formulas)

a. Give for each of the following formulas the set of all sub-formulas of the formula F_i.

$$F_1 = \neg(p_1 \wedge (p_2 \rightarrow \neg p_3))$$
$$F_2 = (p_1 \leftrightarrow \neg p_2) \vee (p_2 \wedge p_1)$$

b. Are the following statements correct? Give a proof.

(a) If all subformulas of F are satisfiable then F is also satisfiable.

(b) A formula F is satisfiable if and only if at least one subformula is satisfiable.

Exercise 1.2 (Equivalences)

a. Use the semantic equivalences presented in the lecture (slide 17) to transform stepwise the following formulas. The resulting formulas have to contain at most the $\wedge$, $\vee$ and $\neg$ connectives (and as few as possible).

(a) $F_1 = \neg\neg\neg(p_1 \vee p_2)$

(b) $F_2 = (p_1 \wedge p_2) \rightarrow p_3$

(c) $F_3 = (p_1 \leftrightarrow \neg p_2) \vee (p_2 \wedge p_1)$

b. Prove that the following statement is true $F \rightarrow G \equiv \neg F \vee G$.

Exercise 1.3 (Truth assignments and models)

Given an assignment α and a formula F, compute F under the assignment α.
Note that in the formulas we often simply use numbers as variables, e.g., 1, 2, 3 instead of x_1, x_2, x_3. Further, we simply identify assignments by a set of literals, e.g., we write $\alpha = \{1, 2, -3\}$ instead of $\alpha = \{1 \mapsto 1, 2 \mapsto 1, 3 \mapsto 0\}$ (or its corresponding interpretation $\{1, 2\}$).

	α	F
a)	$\{1, 2, 3\}$	$(1 \rightarrow -3) \leftrightarrow (2 \wedge -1 \wedge 3)$
b)	$\{-1, 2, 3\}$	
c)	$\{-1, -2, -3, -4, -5\}$	$\Big((-2 \vee 1) \rightarrow \Big(3 \leftrightarrow (5 \vee 2)\Big)\Big) \leftarrow (5 \wedge -3 \wedge 4)$
d)	$\{-1, -2, -3, 4, 5\}$	
e)	$\{1, 2, -3, 4, 5\}$	

Exercise 1.4 (Satisfiability and variants)

Consider the following formula in CNF.

$$F = (\neg x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee x_2) \wedge (x_3 \vee \neg x_4) \wedge (x_2 \vee \neg x_3) \wedge (\neg x_2 \vee \neg x_3 \vee \neg x_5)$$

Solve the following tasks:

a. Decide whether the formula F is satisfiable.

b. Provide a satisfying assignment/model of the formula F (if exists).

c. Compute the number of satisfying assignments/models of the formula F.

d. *Additional Task:* Compute the projected model count when projected to $\{x_4, x_5\}$.
The projected model count pmc of a formula is the number of models when we consider only variables projected to a set P of variables, i.e., $\text{pmc}(F) = \{\, J \cap P \mid J \subseteq \text{var}(F), J \models F \,\}$.

Exercise 1.5 (DPLL Algorithm)

Consider the following formulas.

$$
\begin{aligned}
F_1 = {} & -1 \\
& \wedge (2 \leftrightarrow 1) \\
& \wedge (2 \vee 3) \\
& \wedge (3 \rightarrow -2 \wedge -4) \\
& \wedge (4 \vee 5 \vee 6) \\
& \wedge (5 \rightarrow 7 \wedge 8) \\
& \wedge (-7 \vee 8) \\
& \wedge (5 \leftrightarrow -8)
\end{aligned}
$$

$$
\begin{aligned}
F_2 = {} & (1 \vee -2) \\
& \wedge (1 \rightarrow 3) \\
& \wedge (-3 \vee 2) \\
& \wedge (-2 \vee (4 \wedge 5)) \\
& \wedge (4 \leftrightarrow -3) \\
& \wedge (5 \vee 6 \vee -2) \\
& \wedge (5 \leftrightarrow -6) \\
& \wedge (-1 \rightarrow (3 \vee 6))
\end{aligned}
$$

a. Find a model for F_1 and a model for F_2 (maybe just by looking at the formulas).

b. To each of formulas F_1 and F_2: Apply the DPLL-algorithm and construct a model.

Exercise 1.6 (Resolution and tree-like resolution)

Consider the formula

$$F = (a \vee \neg b \vee d) \wedge (\neg a \vee \neg c \vee \neg d) \wedge (\neg a \vee \neg d) \wedge (b \vee c) \wedge (b \vee \neg e) \wedge (d \vee e) \wedge (\neg d \vee e) \wedge (\neg b \vee \neg e)$$

a. Prove using (Davis-Putnam) resolution that the formula is unsatisfiable.

b. Prove using tree-like resolution (by applying the DPLL-algorithm) that the formula is unsatisfiable.