

**Exercise Sheet 12: Dependencies**  
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**Exercise 12.1.** Let  $\mathcal{L}$  be a fragment of first-order logic for which finite model entailment and arbitrary model entailment coincide, i.e., for every  $\mathcal{L}$ -theory  $\mathcal{T}$  and every  $\mathcal{L}$ -formula  $\varphi$ , we find that  $\varphi$  is true in all models of  $\mathcal{T}$  if and only if  $\varphi$  is true in all finite models of  $\mathcal{T}$ .

- (a) Give an example for a proper fragment of first-order logic with this property.
- (b) Give an example for a proper fragment of first-order logic without this property.
- (c) Show that entailment is decidable in any fragment with this property.

**Exercise 12.2.** Consider the following set of tgds  $\Sigma$ :

$$\begin{aligned}A(x) &\rightarrow \exists y. R(x, y) \wedge B(y) \\B(x) &\rightarrow \exists y. S(x, y) \wedge A(y) \\R(x, y) &\rightarrow S(y, x) \\S(x, y) &\rightarrow R(y, x)\end{aligned}$$

Does the oblivious chase universally terminate for  $\Sigma$ ? What about the restricted chase?

**Exercise 12.3.** Is the following set of tgds  $\Sigma$  weakly acyclic?

$$\begin{aligned}B(x) &\rightarrow \exists y. S(x, y) \wedge A(x) \\A(x) \wedge C(x) &\rightarrow \exists y. R(x, y) \wedge B(y)\end{aligned}$$

Does the skolem chase universally terminate for  $\Sigma$ ?

**Exercise 12.4.** Termination of the oblivious (resp. restricted) chase over a set of tgds  $\Sigma$  implies the existence of a finite universal model for  $\Sigma$ . Is the converse true? That is, does the existence of a finite universal model for  $\Sigma$  imply termination of the oblivious (resp. restricted) chase?

**Exercise 12.5.** Consider a set of tgds  $\Sigma$  that does not contain any constants. A term is *cyclic* if it is of the form  $f(t_1, \dots, t_n)$  and, for some  $i \in \{1, \dots, n\}$ , the function symbol  $f$  syntactically occurs in  $t_i$ . Then  $\Sigma$  is *model-faithful acyclic* (MFA) iff no cyclic term occurs in the skolem chase of  $\Sigma \cup \mathcal{I}_*$ , where  $\mathcal{I}_*$  is the critical instance.

Show the following claims:

1. Checking MFA membership is decidable.
2. Is the set of tgds from Exercise 12.3 MFA?
3. If a set of tgds  $\Sigma$  without constants is MFA, then the skolem chase universally terminates for  $\Sigma$ .