EXERCISE 7

Science of Computational Logic

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Problem 7.1

Give a logic program $\mathcal{P}$ and its completion $C_C(\mathcal{P})$ such that the following holds:

$$\{\neg A \mid \neg A \in C(\mathcal{P})\} \neq \{\neg A \mid \neg A \in C_C(\mathcal{P})\}$$

(Justify your answer.)

Solution

$\mathcal{L}(\{a/0, b/0\}, \mathcal{F}, \mathcal{V})$, i.e. $\mathcal{R} = \{a, b\}$

Let $\mathcal{P} = \{a \leftarrow b\}$.

$\mathcal{R}_D = \{a\} \quad \Rightarrow \quad \mathcal{R} \setminus \mathcal{R}_D = \{b\}$

$C_{\mathcal{P},a} = a \leftarrow b$

$\Rightarrow \quad C_C = \{G \mid \mathcal{P} \cup C_{\mathcal{P},a} \cup \{\neg b\} \models G\}$

We get $\{\neg A \mid \neg A \in C(\mathcal{P})\} = \emptyset$

and $\{\neg A \mid \neg A \in C_C(\mathcal{P})\} \supseteq \{\neg b\}$

Thus $\{\neg A \mid \neg A \in C(\mathcal{P})\} \neq \{\neg A \mid \neg A \in C_C(\mathcal{P})\}$

Problem 7.2

Find non-stratifiable programs K_1 and K_2 such that

- $C_C(K_1)$ is satisfiable, and
- $C_C(K_2)$ is unsatisfiable.

Solution

$$K_1 = \{p \leftarrow \neg q, q \leftarrow \neg p\}$$

$$K_2 = \{p \leftarrow \neg p\}$$

Problem 7.3

Consider the language $\mathcal{L}(\mathcal{R}, \mathcal{F}, \mathcal{V})$ with $\mathcal{R} = \{p/1\}$ and $\mathcal{F} = \{a/0, b/0, c/0\}$.
Let G be the formula $p(a) \wedge (p(b) \vee p(c))$.

- Determine *Circ*(G, p) .

- Find two instantiations G_1 and G_2 of *Circ*(G, p) such that

$$\{G, G_1, G_2\} \models (\forall X)(p(X) \to X \approx a \lor X \approx b) \lor (\forall X)(p(X) \to X \approx a \lor X \approx c).$$

Hint: Combine the ideas from Exercise 1 (slides 23-24) and Exercise 2 (slides 26-27) from the lecture.

Solution

1. *Circ*$(G, p) = (Q(a) \land (Q(b) \lor Q(c)) \land (\forall X)(Q(X) \to p(X))) \to (\forall X)(p(X) \to Q(X))$.

2. We first instantiate *Circ*(G, p) to G_1 by replacing $Q(X)$ by $X \approx a \lor X \approx b$. Note that $Q(a) \land (Q(b) \lor Q(c))$ gets replaced by $(a \approx a \lor a \approx b) \land ((b \approx a \lor b \approx b) \lor (c \approx a \lor c \approx b))$, which is a tautology. So

$$\begin{aligned} G_1 &\equiv (\forall X)(X \approx a \lor X \approx b \to p(X)) \to (\forall X)(p(X) \to X \approx a \lor X \approx b) \\ &\equiv (p(a) \land p(b)) \to (\forall X)(p(X) \to X \approx a \lor X \approx b). \end{aligned}$$

Similary, replacing $Q(X)$ by $X \approx a \lor X \approx c$, we obtain

$$\begin{aligned} G_2 &\equiv (\forall X)(X \approx a \lor X \approx c \to p(X)) \to (\forall X)(p(X) \to X \approx a \lor X \approx c) \\ &\equiv (p(a) \land p(c)) \to (\forall X)(p(X) \to X \approx a \lor X \approx c). \end{aligned}$$

Note that $G \models (p(a) \land p(b)) \lor (p(a) \land p(c))$. So

$$\{G, G_1, G_2\} \models (\forall X)(p(X) \to X \approx a \lor X \approx b) \lor (\forall X)(p(X) \to X \approx a \lor X \approx c).$$