Introduction to Formal Argumentation

* slides adapted from Stefan Woltran’s lecture on Abstract Argumentation

Sarah Gaggl

Dresden, 16th October 2015
Organisation

Learning Outcomes

• The students will get an overview of recent research topics within the field of abstract argumentation
• The students will be able to write a scientific article and give a scientific presentation
• The students will participate in a peer-reviewing process

Organisation:

• 3 introductory lectures
  • Lecture 1: 16.10.2015
  • Lecture 2: 23.10.2015
  • Lecture 3: 30.10.2015
• In last lecture (30.10.2015): article selection
• Students will read related literature and write a seminar paper of 4-5 pages till 4.12.2015
• Each student will review 3 seminar papers from colleagues: 7.12.2015-7.1.2016
• Each student will give a 20 min talk (plus 10 min discussion) about his/her article: 21.-22.1.2016
• Send the slides no later than 1 week before presentation to sarah.gaggl@tu-dresden.de
• Final version of seminar paper are due to 29.01.2015
Roadmap for the Lecture

- Introduction
- Abstract Argumentation Frameworks
  - Syntax
  - Semantics
  - Properties of Semantics
- Implementation Techniques
  - Reduction-based vs. Direct Implementations
  - Reductions to SAT
  - Reductions to ASP
- Generalizations of Abstract Argumentation Frameworks
- Students’ Topics
Introduction

Argumentation:

...the study of processes “concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]
Introduction

**Argumentation:**

...the study of processes “concerned with how assertions are **proposed**, **discussed**, and **resolved** in the context of issues upon which several **diverging opinions** may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

**Formal Models of Argumentation are concerned with**

- representation of an argument
- representation of the relationship between arguments
- solving conflicts between the arguments (“acceptability”)
Introduction (ctd.)

Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: COMMA, TAFA workshop; and several more workshops
- specialized journal: Argument and Computation (Taylor & Francis)
- two text books:
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Handbook of Formal Argumentation HOFA

- [http://formalargumentation.org](http://formalargumentation.org)
- Volume 1 to appear in 2017
### Increasingly important area

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- dedicated conference: **COMMA, TAFA** workshop; and several more workshops
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### Applications

- PARMENIDES-system for E-Democracy: facilitates structured arguments over a proposed course of action [Atkinson et al.; 2006]
- IMPACT project: argumentation toolbox for supporting open, inclusive and transparent deliberations about public policy
- Decision support systems, etc.
- See also [http://comma2014.arg.dundee.ac.uk/demoprogram](http://comma2014.arg.dundee.ac.uk/demoprogram)
The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions
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• Form arguments
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Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]
The Overall Process

Steps

- Starting point: knowledge-base
- **Form arguments**
- Identify conflicts
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Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

\[ \langle \{w, w \rightarrow \neg s\}, \neg s \rangle \]

\[ \langle \{s, s \rightarrow \neg r\}, \neg r \rangle \]

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The Overall Process

Steps

- Starting point: knowledge-base
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Example

\[ \Delta = \{s, r, w, s \to \neg r, r \to \neg w, w \to \neg s\} \]

\[
\begin{align*}
\langle\{w, w \to \neg s\}, \neg s\rangle \\
\langle\{s, s \to \neg r\}, \neg r\rangle & \quad \rightarrow \\
\langle\{r, r \to \neg w\}, \neg w\rangle
\end{align*}
\]
The Overall Process

Steps

- Starting point: knowledge-base
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\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

\[ F_{\Delta} : \]

\[ \alpha \rightarrow \gamma \rightarrow \beta \]
The Overall Process

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- Starting point: knowledge-base
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Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[ F_\Delta : \]

\[ \text{pref}(F_\Delta) = \{ \emptyset \} \]

\[ \text{stage}(F_\Delta) = \{ \{ \alpha \}, \{ \beta \}, \{ \gamma \} \} \]

TU Dresden, 16th October 2015
Seminar Abstract Argumentation
slide 14 of 95
The Overall Process

Steps
- Starting point: knowledge-base
- Form arguments
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- Draw conclusions

Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[ Cn_{\text{pref}}(F_{\Delta}) = Cn(\top) \]

\[ Cn_{\text{stage}}(F_{\Delta}) = Cn(\neg r \lor \neg w \lor \neg s) \]
## The Overall Process (ctd.)

### Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")
Some Remarks

• Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)

• Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")

• Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")

Main Challenge

• All Steps in the argumentation process are, in general, intractable.

• This calls for:
  • careful complexity analysis (identification of tractable fragments)
  • re-use of established tools for implementations (reduction method)
Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) $\Delta$
- argument is a pair $(\Phi, \alpha)$, such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
- conflicts between arguments $(\Phi, \alpha)$ and $(\Phi', \alpha')$ arise if $\Phi$ and $\alpha'$ are contradicting.
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Example

\[ \langle \{s, s\rightarrow \neg r\}, \neg r \rangle \rightarrow \langle \{r, r\rightarrow \neg w\}, \neg w \rangle \]
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Example

$\langle \{s, s \rightarrow \neg r\}, \neg r \rangle \rightarrow \langle \{r, r \rightarrow \neg w\}, \neg w \rangle$

Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.
Dung’s Abstract Argumentation Frameworks

Example

\[ \begin{array}{c}
\alpha \\
\gamma \\
\beta 
\end{array} \]

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.

"plethora of semantics"
Dung’s Abstract Argumentation Frameworks

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**Definition**

An **argumentation framework** (AF) is a pair \((A, R)\) where

- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing the conflicts (“attacks”)
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**Example**

\[
F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\})
\]
## Basic Properties

### Conflict-Free Sets

Given an AF \( F = (A, R) \).

A set \( S \subseteq A \) is **conflict-free** in \( F \), if, for each \( a, b \in S \), \( (a, b) \notin R \).
Basic Properties

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Example

$$cf(F) = \{a, c\},$$
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Example

\[
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TU Dresden, 16th October 2015 Seminar Abstract Argumentation slide 33 of 95
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Example

$\text{adm}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Basic Properties (ctd.)

**Dung’s Fundamental Lemma**

Let $S$ be admissible in an AF $F$ and $a, a'$ arguments in $F$ defended by $S$ in $F$. Then,

1. $S' = S \cup \{a\}$ is admissible in $F$
2. $a'$ is defended by $S'$ in $F$
Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of $F$, if

- $S$ is conflict-free in $F$,
- for each $T \subseteq A$ conflict-free in $F$, $S \not\subset T$
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Example

\[ \text{naive}(F) = \{ \{a, c\}, \{a, d\} \}, \]
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naive(F) = \{ \{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset \}
\]
Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of $F$ is defined as the outcome $S$ of the following “algorithm”:

1. put each argument $a \in A$ which is not attacked in $F$ into $S$; if no such argument exists, return $S$;
2. remove from $F$ all (new) arguments in $S$ and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.
Semantics (ctd.)

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Example

$ground(F) = \{ \{a\} \}$
Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)
  - Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).
Semantics (ctd.)

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Example

\[
\begin{align*}
& a \quad b \quad c \quad d \quad e \\
\end{align*}
\]

\(\text{comp}(F) = \{a, c\}\)
Semantics (ctd.)

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Example

\[
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Example

\[\text{comp}(F) = \{\{a, c\}, \{a, d\}, \{a\}\},\]
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**Example**

\[
\begin{align*}
\text{comp}(F) & = \{\{a, c\}, \{a, d\}, \{a\}, \{e\}, \{d\}, \emptyset\} \\
\end{align*}
\]
Properties of the Grounded Extension

For any AF $F$, the grounded extension of $F$ is the subset-minimal complete extension of $F$. 

Remark: Since there exists exactly one grounded extension for each AF $F$, we often write $\text{ground}(F) = S$ instead of $\text{ground}(F) = \{S\}$. 
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Preferred Extensions [Dung, 1995]

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Example

$$\text{pref}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$$
Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$
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Example

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\text{stable}(F) = \{\{a, e\}\}
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Example

$$\text{stable}(F) = \{ \{a, e\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{e\}, \{d\}, \emptyset \}$$
Some Relations

For any AF $F$ the following relations hold:

1. Each stable extension of $F$ is admissible in $F$
2. Each stable extension of $F$ is also a preferred one
3. Each preferred extension of $F$ is also a complete one
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S^+ \not\subseteq T^+$

- for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$
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**Example**

$$semi(F) = \{\{a, e\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$$
Semantics (ctd.)

Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S^+ \not\subseteq T^+$
  - recall $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

Ideal Extension [Dung, Mancarella & Toni 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of $F$, if

- $S$ is admissible in $F$ and contained in each preferred extension of $F$
- there is no $T \supset S$ admissible in $F$ and contained in each of $\text{pref}(F)$

Eager Extension [Caminada, 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an eager extension of $F$, if

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**Eager Extension [Caminada, 2007]**

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- there is no $T \supset S$ admissible in $F$ and contained in each of $\text{semi}(F)$
Properties of Ideal Extensions

For any AF $F$ the following observations hold:

1. there exists exactly one ideal extension of $F$
2. the ideal extension of $F$ is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].
Resolution-based grounded Extensions
[Baroni,Giacomin 2008]

A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b), (b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of $F$, if
- there exists a resolution $\beta$ such that $\text{ground}((A, R \setminus \beta)) = S$
- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$
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**Example**

$\text{ground}^*(F) = \{\{a, c\}\}$,
A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b), (b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

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Example

$\text{ground}^*(F) = \{\{a, c\}, \{a, d\}\}$
Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of $F$.

Example
Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of $F$.

Example
Definition (Reachability)

Let $F = (A, R)$ be an AF, $B$ a set of arguments, and $a, b \in A$. We say that $b$ is reachable in $F$ from $a$ modulo $B$, in symbols $a \Rightarrow^B_F b$, if there exists a path from $a$ to $b$ in $F|_B$. 
**Definition (Reachability)**

Let $F = (A, R)$ be an AF, $B$ a set of arguments, and $a, b ∈ A$. We say that $b$ is **reachable** in $F$ from $a$ **modulo** $B$, in symbols $a \Rightarrow^B_F b$, if there exists a path from $a$ to $b$ in $F|_B$.

**Definition ($\Delta_{F,S}$)**

For an AF $F = (A, R)$, $D \subseteq A$, and a set $S$ of arguments,

$$\Delta_{F,S}(D) = \{a ∈ A \mid ∃b ∈ S : b ≠ a, (b, a) ∈ R, a \not\Rightarrow^A \setminus D_F b\}.$$  

By $\Delta_{F,S}$, we denote the lfp of $\Delta_{F,S}(∅)$. 
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in \text{naive}([[F - \Delta_{F,S}]]).$
cf2 Extensions [G & Woltran 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([F - \Delta_{F,S}])$.

Example

$S = \{c, f, h\}$, $S \in cf(F)$. 

TU Dresden, 16th October 2015 Seminar Abstract Argumentation slide 72 of 95
cf2 Semantics (ctd.)

**cf2 Extensions [G & Woltran 2010]**

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a **cf2-extension** of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([F - \Delta_{F,S}])$.

**Example**

$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S}(\emptyset) = \{d, e\}$.

---

TU Dresden, 16th October 2015  Seminar Abstract Argumentation  slide 73 of 95
cf2 Extensions [G & Woltran 2010]

Given an AF \( F = (A, R) \). A set \( S \subseteq A \) is a cf2-extension of \( F \), if
- \( S \) is conflict-free in \( F \)
- and \( S \in naive([[F - \Delta_{F,S}]])) \).

Example

\( S = \{c,f,h\}, \ S \in cf(F), \ \Delta_{F,S}(\{d,e\}) = \{d,e\} \).
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([F - \Delta_{F,S}])$.

**Example**

$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S} = \{d, e\}$, $S \in naive([F - \Delta_{F,S}])$. 
Relations between Semantics

Figure: An arrow from semantics $\sigma$ to semantics $\tau$ encodes that each $\sigma$-extension is also a $\tau$-extension.
Characteristics of Argumentation Semantics

Example

\[
\text{pref}(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}
\]

\[
\text{naive}(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\}
\]

Natural Questions

- How to change the AF if we want \(\{a, b, e\}\) instead of \(\{a, b\}\) in \(\text{pref}(F)\)?
- How to change the AF if we want \(\{a, b, d\}\) instead of \(\{a, b\}\) in \(\text{pref}(F)\)?
- Can we have equivalent AFs without argument \(f\)?

→ Realizability
Some Properties . . .

**Theorem**

For any AFs $F$ and $G$, we have

- $\text{adm}(F) = \text{adm}(G) \implies \sigma(F) = \sigma(G)$, for $\sigma \in \{\text{pref}, \text{ideal}\}$;
- $\text{comp}(F) = \text{comp}(G) \implies \vartheta(F) = \vartheta(G)$, for $\vartheta \in \{\text{pref}, \text{ideal}, \text{ground}\}$;
- no other such relation between the different semantics ($\text{adm}$, $\text{pref}$, $\text{ideal}$, $\text{semi}$, $\text{eager}$, $\text{ground}$, $\text{comp}$, $\text{stable}$) in terms of standard equivalence holds.
Strong Equivalence [Oikarinen & Woltran 2011, G & Woltran 2011]

**Definition**

Two AFs $F$ and $G$ are strongly equivalent wrt. a semantics $\sigma \in \{\text{stable, adm, pref, ideal, semi, comp, ground, stage}\}$, in symbols $F \equiv^\sigma_s G$, iff $\sigma(F \cup H) = \sigma(G \cup H)$, for each AF $H$.

- Idea: Find “$\sigma$-kernels” of AFs, such that the $\sigma$-kernels of $F$ and $G$ coincide iff $F \equiv^\sigma_s G$.
- Verification of strong equivalence then reduces to checking syntactical equivalence.
Kernel for stable semantics

For AF $F = (A, R)$, we define stable-kernel of $F$ as $F^\kappa = (A, R^\kappa)$ with

$$R^\kappa = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}.$$ 

Theorem

For any AFs $F$ and $G$: $F^\kappa = G^\kappa$ iff $F \equiv_s^{stable} G$ iff $F \equiv_s^{stage} G$. 
### Credulous Acceptance

\[ \text{Cred}_\sigma : \text{Given AF } F = (A, R) \text{ and } a \in A; \text{ is } a \text{ contained in at least one } \sigma\text{-extension of } F? \]

### Skeptical Acceptance

\[ \text{Skept}_\sigma : \text{Given AF } F = (A, R) \text{ and } a \in A; \text{ is } a \text{ contained in every } \sigma\text{-extension of } F? \]

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted\(^1\).

\(^1\) This is only relevant for stable semantics.
Credulous Acceptance

\[ \text{Cred}_\sigma : \text{Given AF } F = (A, R) \text{ and } a \in A; \text{ is } a \text{ contained in at least one } \sigma\text{-extension of } F? \]

Skeptical Acceptance

\[ \text{Skept}_\sigma : \text{Given AF } F = (A, R) \text{ and } a \in A; \text{ is } a \text{ contained in every } \sigma\text{-extension of } F? \]

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted\(^1\).

Hence we are also interested in the following problem:

Skeptically and Credulously accepted

\[ \text{Skept}'_\sigma : \text{Given AF } F = (A, R) \text{ and } a \in A; \text{ is } a \text{ contained in every and at least one } \sigma\text{-extension of } F? \]

---

\(^1\)This is only relevant for stable semantics.
Further Decision Problems

Verifying an extension

$\text{Ver}_\sigma$: Given $AF F = (A, R)$ and $S \subseteq A$; is $S$ a $\sigma$-extension of $F$?
## Further Decision Problems

### Verifying an extension

**Ver}_ \sigma: Given AF \( F = (A, R) \) and \( S \subseteq A \); is \( S \) a \( \sigma \)-extension of \( F \)?

### Does there exist an extension?

**Exists}_ \sigma: Given AF \( F = (A, R) \); Does there exist a \( \sigma \)-extension for \( F \)?
### Further Decision Problems

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<th>Verifying an extension</th>
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<td>( \text{Ver}_\sigma : \text{Given } \mathcal{AF} \ F = (A, R) \text{ and } S \subseteq A; \text{ is } S \text{ a } \sigma\text{-extension of } F? )</td>
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<th>Does there exist an extension?</th>
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<td>( \text{Exists}_\sigma : \text{Given } \mathcal{AF} \ F = (A, R); \text{ Does there exist a } \sigma\text{-extension for } F? )</td>
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<th>Does there exist a nonempty extension?</th>
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<td>( \text{Exists}_{\neg \emptyset} : \text{Does there exist a non-empty } \sigma\text{-extension for } F? )</td>
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Complexity for decision problems in AFs.

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<tr>
<th>$\sigma$</th>
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<tr>
<td>ground</td>
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<td>$\Sigma_2^p$-c</td>
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<td>naive</td>
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<td>$\Sigma_2^p$-c</td>
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<td>co-NP-c</td>
<td>ideal</td>
<td>in $\Theta_2^p$</td>
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<tr>
<td>adm</td>
<td>NP-c</td>
<td>trivial</td>
<td>eager</td>
<td>$\Pi_2^p$-c</td>
<td>$\Pi_2^p$-c</td>
</tr>
<tr>
<td>comp</td>
<td>NP-c</td>
<td>P-c</td>
<td>ground*</td>
<td>NP-c</td>
<td>co-NP-c</td>
</tr>
<tr>
<td>pref</td>
<td>NP-c</td>
<td>$\Pi_2^p$-c</td>
<td>cf2</td>
<td>NP-c</td>
<td>co-NP-c</td>
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Intractable problems in Abstract Argumentation

Most problems in Abstract Argumentation are computationally intractable, i.e. at least NP-hard. To show intractability for a specific reasoning problem we follow the schema given below:

**Goal:** Show that a reasoning problem is NP-hard.

**Method:** Reducing the NP-hard SAT problem to the reasoning problem.

- Consider an arbitrary CNF formula $\varphi$
- Give a reduction that maps $\varphi$ to an Argumentation Framework $F_\varphi$ containing an argument $\varphi$.
- Show that $\varphi$ is satisfiable iff the argument $\varphi$ is accepted.
Canonical Reduction

Definition

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \lor l_{i2} \lor l_{i3}$ over atoms $Z$, build $F_{\varphi} = (A_{\varphi}, R_{\varphi})$ with

$A_{\varphi} = Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\}$

$R_{\varphi} = \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}$
Canonical Reduction

**Definition**

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \lor l_{i2} \lor l_{i3}$ over atoms $Z$, build $F_{\varphi} = (A_{\varphi}, R_{\varphi})$ with

\[
A_{\varphi} = Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\}
\]

\[
R_{\varphi} = \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \\
\{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \\
\{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}
\]

**Example**

Let $\Phi = (z_1 \lor z_2 \lor z_3) \land (\neg z_2 \lor \neg z_3 \lor \neg z_4) \land (\neg z_1 \lor z_2 \lor z_4)$.
Theorem

The following statements are equivalent:

1. \( \varphi \) is satisfiable
2. \( F\varphi \) has an admissible set containing \( \varphi \)
3. \( F\varphi \) has a complete extension containing \( \varphi \)
4. \( F\varphi \) has a preferred extension containing \( \varphi \)
5. \( F\varphi \) has a stable extension containing \( \varphi \)
Complexity Results

Theorem

1. $\text{Cred}_{\text{stable}}$ is NP-complete
2. $\text{Cred}_{\text{adm}}$ is NP-complete
3. $\text{Cred}_{\text{comp}}$ is NP-complete
4. $\text{Cred}_{\text{pref}}$ is NP-complete

Proof.

(1) The hardness is immediate by the last theorem. For the NP-membership we use the following guess & check algorithm:

- Guess a set $E \subseteq A$
- verify that $E$ is stable
  - for each $a, b \in E$ check $(a, b) \not\in R$
  - for each $a \in A \setminus E$ check if there exists $b \in E$ with $(b, a) \in R$

As this algorithm is in polynomial time we obtain NP-membership.

TU Dresden, 16th October 2015
Seminar Abstract Argumentation
slide 91 of 95
Summary

What did we learn today?

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P. Baroni and M. Giacomin.
Semantics of abstract argument systems.


T.J.M. Bench-Capon and P.E.Dunne.
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S. Coste-Marquis, C. Devred, and P. Marquis.
Symmetric argumentation frameworks.

Y. Dimopoulos and A. Torres.
Graph theoretical structures in logic programs and default theories.

P. M. Dung.
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Coherence in finite argument systems.

P. E. Dunne and T. J. M. Bench-Capon.
Complexity in value-based argument systems.

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Parametric properties of ideal semantics.

W. Dvořák and S. Woltran
On the intertranslatability of argumentation semantics

S. Gaggl and S. Woltran.
cf2 semantics revisited.

S. Gaggl and S. Woltran.
Strong equivalence for argumentation semantics based on conflict-free sets.

E. Oikarinen and S. Woltran.
Characterizing strong equivalence for argumentation frameworks.

B. Verheij.
Two approaches to dialectical argumentation: admissible sets and argumentation stages.