TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

SEMINAR ABSTRACT ARGUMENTATION

Introduction to Formal Argumentation

*slides adapted from Stefan Woltran's lecture on Abstract Argumentation

Sarah Gaggl

Dresden, 16th October 2015

Organisation

Learning Outcomes

- The students will get an overview of recent research topics within the field of abstract argumentation
- The students will be able to write a scientific article and give a scientific presentation
- The students will participate in a peer-reviewing process

Organisation:

- 3 introductory lectures
  - Lecture 1: 16.10.2015
  - Lecture 2: 23.10.2015
  - Lecture 3: 30.10.2015
- In last lecture (30.10.2015): article selection
- Students will read related literature and write a seminar paper of 4-5 pages till 4.12.2015
- Each student will review 3 seminar papers from colleagues: 7.12.2015-7.1.2016
- Each student will give a 20 min talk (plus 10 min discussion) about his/her article: 21.-22.1.2016
- Send the slides no later than 1 week before presentation to sarah.gaggl@tu-dresden.de
- Final version of seminar paper are due to 29.01.2015

Roadmap for the Lecture

- Introduction
- Abstract Argumentation Frameworks
  - Syntax
  - Semantics
  - Properties of Semantics
- Implementation Techniques
  - Reduction-based vs. Direct Implementations
  - Reductions to SAT
  - Reductions to ASP
- Generalizations of Abstract Argumentation Frameworks
- Students' Topics

Introduction

Argumentation:

. . . the study of processes "concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held".

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

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Formal Models of Argumentation are concerned with

- representation of an argument
- representation of the relationship between arguments
- solving conflicts between the arguments ("acceptability")

Introduction (ctd.)

Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: COMMA, TAFA workshop; and several more workshops
- specialized journal: Argument and Computation (Taylor & Francis)
- two text books:
  - Besnard, Hunter: Elements of Argumentation. MIT Press, 2008
  - Rahwan, Simari (eds.): Argumentation in Artificial Intelligence. Springer, 2009.

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Handbook of Formal Argumentation HOFA

- `http://formalargumentation.org`
- Volume 1 to appear in 2017

Introduction (ctd.)

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Applications

- PARMENIDES-system for E-Democracy: facilitates structured arguments over a proposed course of action [Atkinson et al.; 2006]
- IMPACT project: argumentation toolbox for supporting open, inclusive and transparent deliberations about public policy
- Decision support systems, etc.
- See also `http://comma2014.arg.dundee.ac.uk/demoprogram`.

The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

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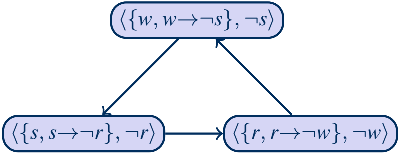

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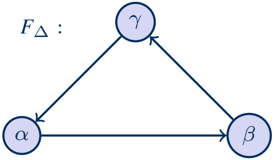
F_Δ :



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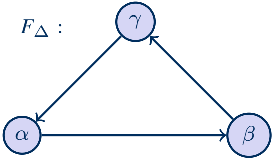

$$pref(F_\Delta) = \{\emptyset\}$$
$$stage(F_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$$

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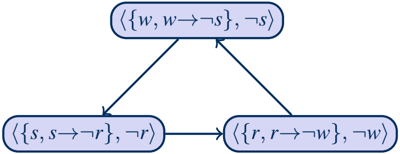

$$Cn_{pref}(F_\Delta) = Cn(\top)$$
$$Cn_{stage}(F_\Delta) = Cn(\neg r \vee \neg w \vee \neg s)$$

The Overall Process (ctd.)

Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")

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Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) Δ
- argument is a pair (Φ, α), such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
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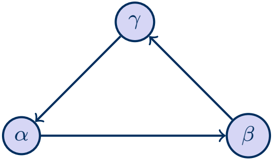
Example

Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

Dung's Abstract Argumentation Frameworks

Example



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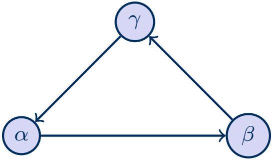

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - "plethora of semantics"

Dung's Abstract Argumentation Frameworks

Definition

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

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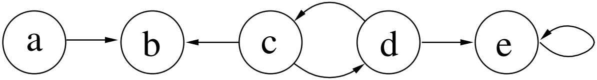
Example

$F = (\{a, b, c, d, e\} \, , \, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\} \,)$



Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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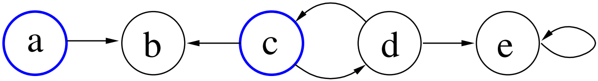

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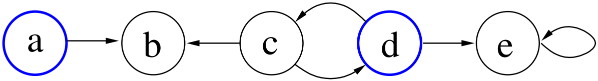

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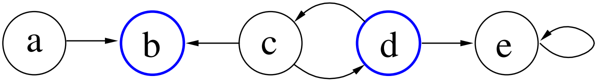

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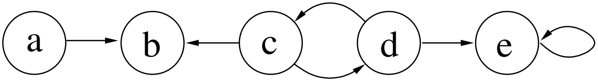

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Basic Properties (ctd.)

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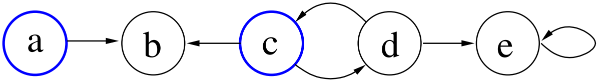

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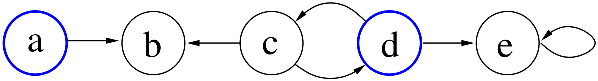

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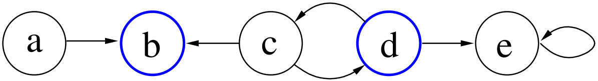

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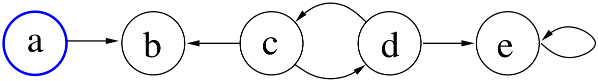

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Basic Properties (ctd.)

Dung's Fundamental Lemma

Let S be admissible in an AF F and a, a' arguments in F defended by S in F. Then,

1. $S' = S \cup \{a\}$ is admissible in F
2. a' is defended by S' in F

Semantics

Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of F, if

- S is conflict-free in F
- for each $T \subseteq A$ conflict-free in F, $S \not\subset T$

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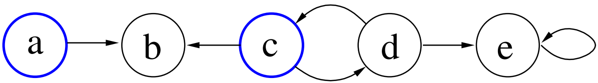

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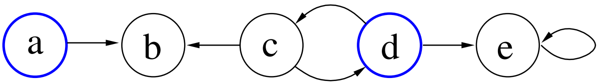

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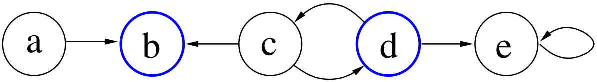

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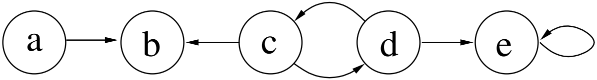

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Semantics (ctd.)

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of F is defined as the outcome S of the following "algorithm":

1. put each argument $a \in A$ which is not attacked in F into S; if no such argument exists, return S;
2. remove from F all (new) arguments in S and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

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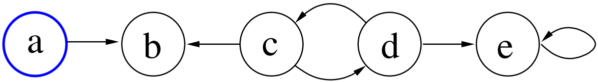

$ground(F) = \{\{a\}\}$

Semantics (ctd.)

Complete Extension [Dung, 1995]

Given an AF (A, R). A set $S \subseteq A$ is complete in F, if

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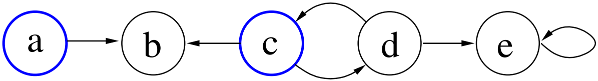

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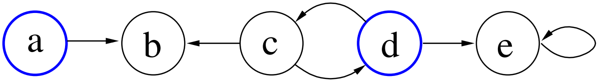

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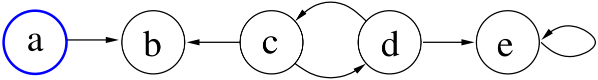

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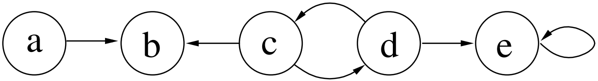

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Semantics (ctd.)

Properties of the Grounded Extension

For any AF F, the grounded extension of F is the subset-minimal complete extension of F.

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Remark

Since there exists exactly one grounded extension for each AF F, we often write $ground(F) = S$ instead of $ground(F) = \{S\}$.

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Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

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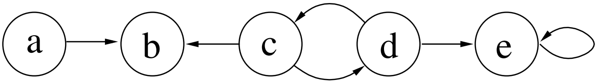
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S \not\subset T$

Example



$pref(F) = \{\{a, c\}, \{a, d\}, ~~\{a\}, \{c\}, \{d\}, \emptyset~~\}$

Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Semantics (ctd.)

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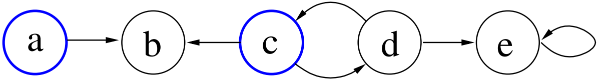

$stable(F) = \{\sout{\{a, c\}}\}$

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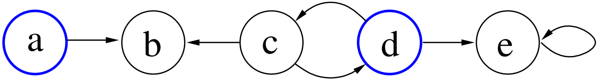

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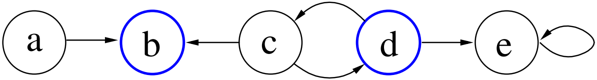

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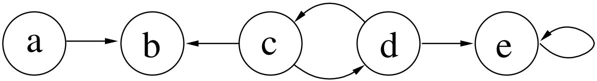

$stable(F) = \{$ ~~$\{a, c\}$~~, $\{a, d\}$, ~~$\{b, d\}$~~, ~~$\{a\}$~~, ~~$\{b\}$~~, ~~$\{c\}$~~, ~~$\{d\}$~~, ~~$\emptyset$~~, $\}$

Semantics (ctd.)

Some Relations

For any AF F the following relations hold:

1. Each stable extension of F is admissible in F
2. Each stable extension of F is also a preferred one
3. Each preferred extension of F is also a complete one

Semantics (ctd.)

Semi-Stable Extensions [Caminada, 2006]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in F, $S^+ \not\subset T^+$
  - for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

Semantics (ctd.)

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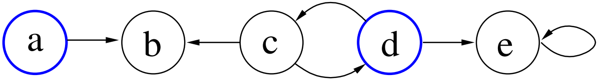

$semi(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{a\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$

Semantics (ctd.)

Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of F, if

- S is conflict-free in F
- for each $T \subseteq A$ conflict-free in F, $S^+ \not\subset T^+$
    - recall $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

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    - recall $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

Ideal Extension [Dung, Mancarella & Toni 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of F, if

- S is admissible in F and contained in each preferred extension of F
- there is no $T \supset S$ admissible in F and contained in each of $\mathit{pref}(F)$

Semantics (ctd.)

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Eager Extension [Caminada, 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an eager extension of F, if

- S is admissible in F and contained in each semi-stable extension of F
- there is no $T \supset S$ admissible in F and contained in each of $semi(F)$

Semantics (ctd.)

Properties of Ideal Extensions

For any AF F the following observations hold:

1. there exists exactly one ideal extension of F
2. the ideal extension of F is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].

Semantics (ctd.)

Resolution-based grounded Extensions [Baroni,Giacomin 2008]

A resolution β of an AF $F = (A, R)$ contains exactly one of the attacks (a, b), (b, a) for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of F, if

- there exists a resolution β such that $ground((A, R \setminus \beta)) = S$
- and there is no resolution β' such that $ground((A, R \setminus \beta')) \subset S$

Semantics (ctd.)

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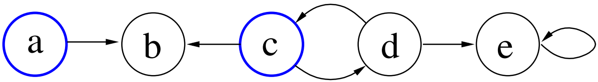

$ground^*(F) = \{\{a, c\},$

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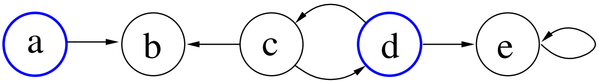


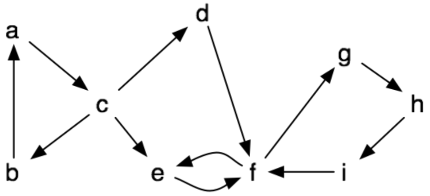
$ground^*(F) = \{\{a, c\}, \{a, d\}\}$

cf2 Semantics [Baroni, Giacomin & Guida 2005]

Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from b to a. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of F.

Example



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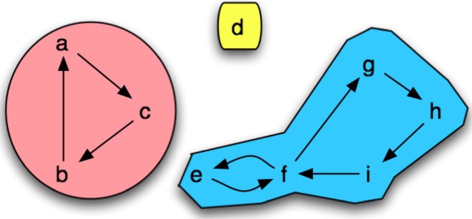

cf2 Semantics (ctd.)

Definition (Reachability)

Let $F = (A, R)$ be an AF, B a set of arguments, and $a, b \in A$. We say that b is reachable in F from a modulo B, in symbols $a \Rightarrow_F^B b$, if there exists a path from a to b in $F|_B$.

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Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set S of arguments,

$$\Delta_{F,S}(D) = \{a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow_F^{A \setminus D} b\}.$$

By $\Delta_{F,S}$, we denote the lfp of $\Delta_{F,S}(\emptyset)$.

cf2 Semantics (ctd.)

cf2 Extensions [G & Woltran 2010]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of F, if

- S is conflict-free in F
- and $S \in naive([[F - \Delta_{F,S}]])$.

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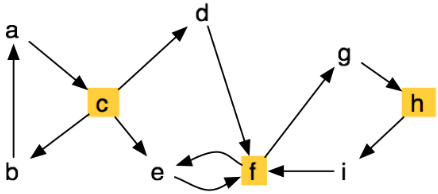
Example

$S = \{c, f, h\}$, $S \in cf(F)$.



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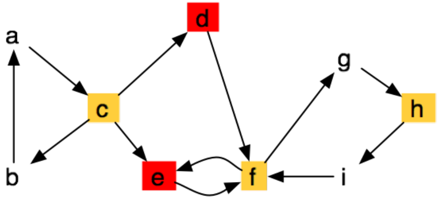
$S = \{c, f, h\},\ S \in cf(F),\ \Delta_{F,S}(\emptyset) = \{d, e\}$.



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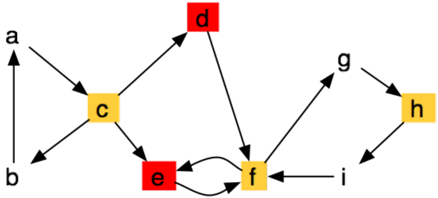

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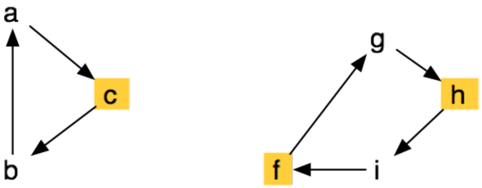
Example

$S = \{c, f, h\},\ S \in cf(F),\quad \Delta_{F,S} = \{d, e\}, S \in naive([[F - \Delta_{F,S}]])$.



Relations between Semantics

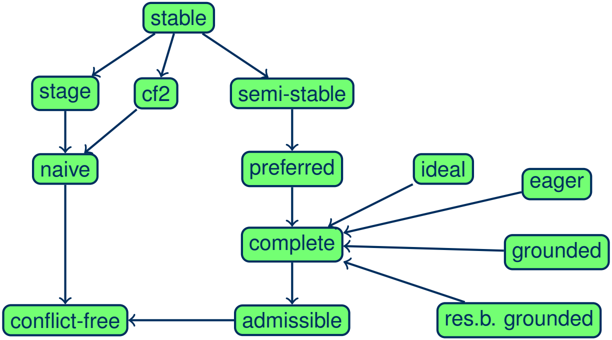
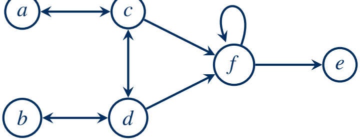


Figure : An arrow from semantics σ to semantics τ encodes that each σ-extension is also a τ-extension.

Characteristics of Argumentation Semantics

Example



$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$
$naive(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\}$

Natural Questions

- How to change the AF if we want $\{a, b, e\}$ instead of $\{a, b\}$ in $pref(F)$?
- How to change the AF if we want $\{a, b, d\}$ instead of $\{a, b\}$ in $pref(F)$?
- Can we have equivalent AFs without argument f?

→ Realizability

Some Properties . . .

Theorem

For any AFs F and G, we have

- $adm(F) = adm(G) \Longrightarrow \sigma(F) = \sigma(G)$, for $\sigma \in \{pref, ideal\}$;
- $comp(F) = comp(G) \Longrightarrow \vartheta(F) = \vartheta(G)$, for $\vartheta \in \{pref, ideal, ground\}$;
- no other such relation between the different semantics (*adm*, *pref*, *ideal*, *semi*, *eager*, *ground*, *comp*, *stable*) in terms of standard equivalence holds.

Strong Equivalence [Oikarinen & Woltran 2011, G & Woltran 2011]

Definition

Two AFs F and G are strongly equivalent wrt. a semantics $\sigma \in \{stable, adm, pref, ideal, semi, comp, ground, stage\}$, in symbols $F \equiv_s^\sigma G$, iff $\sigma(F \cup H) = \sigma(G \cup H)$, for each AF H.

- Idea: Find "σ-kernels" of AFs, such that the σ-kernels of F and G coincide iff $F \equiv_s^\sigma G$.
  - Verification of strong equivalence then reduces to checking syntactical equivalence

Strong Equivalence for Stable Semantics

Kernel for stable semantics

For AF $F = (A, R)$, we define *stable*-kernel of F as $F^{\kappa} = (A, R^{\kappa})$ with

$$R^{\kappa} = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}.$$

Theorem

For any AFs F and G: $F^{\kappa} = G^{\kappa}$ iff $F \equiv_s^{stable} G$ iff $F \equiv_s^{stage} G$.

Decision Problems on AFs

Credulous Acceptance

Cred$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in at least one σ-extension of F?

Skeptical Acceptance

Skept$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in every σ-extension of F?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted[1].

[1] This is only relevant for stable semantics.

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Hence we are also interested in the following problem:

Skeptically and Credulously accepted

Skept$'_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is a contained in every and at least one σ-extension of F?

---

[1] This is only relevant for stable semantics.

Further Decision Problems

Verifying an extension

Ver$_\sigma$: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Further Decision Problems

Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Does there exist an extension?

Exists_σ: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Further Decision Problems

Verifying an extension

Ver$_\sigma$: Given AF $F = (A, R)$ and $S \subseteq A$; is S a σ-extension of F?

Does there exist an extension?

Exists$_\sigma$: Given AF $F = (A, R)$; Does there exist a σ-extension for F?

Does there exist a nonempty extensions?

Exists$_\sigma^{\neg\emptyset}$: Does there exist a non-empty σ-extension for F?

Complexity Results (Summary)

Complexity for decision problems in AFs.

σ	Cred_σ	Skept_σ
ground	P-c	P-c
naive	in L	in L
stable	NP-c	co-NP-c
adm	NP-c	trivial
comp	NP-c	P-c
pref	NP-c	Π_2^p-c

σ	Cred_σ	Skept_σ
semi	Σ_2^p-c	Π_2^p-c
stage	Σ_2^p-c	Π_2^p-c
ideal	in Θ_2^p	in Θ_2^p
eager	Π_2^p-c	Π_2^p-c
*ground**	NP-c	co-NP-c
cf2	NP-c	co-NP-c

see [Baroni et al.2011, Coste-Marquis et al.2005, Dimopoulos and Torres1996, Dung1995, Dunne2008, Dunne and Bench-Capon2002, Dunne and Bench-Capon2004, Dunne and Caminada2008, Dvořák et al.2011, Dvořák and Woltran2010a, Dvořák and Woltran2010b]

Intractable problems in Abstract Argumentation

Most problems in Abstract Argumentation are computationally intractable, i.e. at least NP-hard. To show intractability for a specific reasoning problem we follow the schema given below:

Goal: Show that a reasoning problem is NP-hard.

Method: Reducing the NP-hard SAT problem to the reasoning problem.

- Consider an arbitrary CNF formula φ
- Give a reduction that maps φ to an Argumentation Framework F_φ containing an argument φ.
- Show that φ is satisfiable iff the argument φ is accepted.

Canonical Reduction

Definition

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \vee l_{i2} \vee l_{i3}$ over atoms Z, build $F_\varphi = (A_\varphi, R_\varphi)$ with

$$\begin{aligned}
A_\varphi &= Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\} \\
R_\varphi &= \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \\
&\quad \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \\
&\quad \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}
\end{aligned}$$

Canonical Reduction

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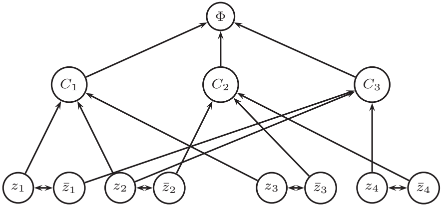
$$\begin{aligned} A_\varphi &= Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\} \\ R_\varphi &= \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \\ &\quad \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \\ &\quad \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \end{aligned}$$

Example

Let $\Phi = (z_1 \vee z_2 \vee z_3) \wedge (\neg z_2 \vee \neg z_3 \vee \neg z_4) \wedge (\neg z_1 \vee z_2 \vee z_4)$.



Canonical Reduction: CNF $\Rightarrow$ AF (ctd.)

Theorem

The following statements are equivalent:

1. φ is satisfiable
2. F_φ has an admissible set containing φ
3. F_φ has a complete extension containing φ
4. F_φ has a preferred extension containing φ
5. F_φ has a stable extension containing φ

Complexity Results

Theorem

1. Cred_{stable} is NP-complete
2. Cred_{adm} is NP-complete
3. Cred_{comp} is NP-complete
4. Cred_{pref} is NP-complete

Proof.

(1) The hardness is immediate by the last theorem.
For the NP-membership we use the following guess & check algorithm:

- Guess a set $E \subseteq A$
- verify that E is stable
  - for each $a, b \in E$ check $(a, b) \notin R$
  - for each $a \in A \setminus E$ check if there exists $b \in E$ with $(b, a) \in R$

As this algorithm is in polynomial time we obtain NP-membership. □

Summary

What did we learn today?

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-
-

⋮

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