

# Human Reasoning and Computational Logic

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## Problem 3.1

Given the following definite program  $\mathcal{P}$ :

$$\begin{aligned} & q(X, X, c) \\ q(f(X), Y, f(Z)) & \leftarrow q(X, Y, Z) \\ & p(X, X, f(c)) \\ p(X, Y, f(Z)) & \leftarrow p(X', Y, Z) \wedge q(X, X', Y) \end{aligned}$$

1. Compute  $T_{\mathcal{P}}^n(\emptyset)$  for  $n = 1, \dots, 4$ .
2. Find the least Herbrand model of the program  $\mathcal{P}$ .

## Problem 3.2

Consider the following logic program  $\mathcal{P}$ :

$$\begin{aligned} q(X) & \leftarrow \\ p(a) & \leftarrow p(a) \\ p(b) & \leftarrow q(X) \\ p(s(X)) & \leftarrow p(X) \end{aligned}$$

1. Let  $I_0 = \{p(a)\}$  and  $I_{n+1} = T_{\mathcal{P}}(I_n)$  for all  $n \in \mathbb{N}$ . Compute  $I_n$  for all  $n \in \mathbb{N}$ .
2. Show that  $\bigcup_{n \in \mathbb{N}} I_n$  is not the least Herbrand model of  $\mathcal{P}$ .

## Problem 3.3

Given the following program  $\mathcal{P}$ :

$$\begin{aligned} & p(a, X, X) \\ p(s(X), Y, s(Z)) & \leftarrow p(X, Y, Z) \end{aligned}$$

Compute  $T_{\mathcal{P}} \uparrow n$  for every  $n \in \mathbb{N}$  and compute  $\text{lfp}(T_{\mathcal{P}})$ .

## Problem 3.4

Given a definite program  $\mathcal{P}$ . Show that  $T_{\mathcal{P}}$  is monotonic.