

Knowledge Representation and Reasoning

Horn Logics and Datalog Problems

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Problem 1. Consider the following set of formulae:

1. $A \vee \neg A$
2. $(\neg A \vee B) \wedge A \wedge (\neg C \vee A \vee B) \wedge \neg B$
3. $(A \vee B) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

Decide which of these formulae are Horn-formulae.

Problem 2. Consider the following knowledge bases (1) in propositional logic and (2) in first-order logic:

Knowledge Base (1):

- $Cinema \vee SwimmingPool$
- $Cinema \rightarrow Fun$
- $SwimmingPool \vee Sleeping_In \rightarrow Fun$

Knowledge Base (2):

- $\forall x GradStudent(x) \rightarrow Student(x)$
- $\forall x Student(x) \rightarrow HardWorker(x)$
- $GradStudent(Lisa)$

Use Resolution to:

1. Show $KB_1 \models Fun$.
2. Show $KB_1 \not\models SwimmingPool$.

3. Decide whether $KB_2 \models \text{HardWorker}(\text{Lisa})$ holds.

Problem 3. Assume that $\text{Parent}(x, y)$ is true iff (if and only if) x is a parent of y , $\text{Male}(x)$ is true iff x is male and $\text{Female}(x)$ is true iff x is female. Write down Datalog rules that specify membership conditions for the binary familial relationship predicates Father , Mother , Son , Daughter , Sibling , Brother , Sister , Grandparent , Uncle , and Cousin . For example:

$$\text{Parent}(x, y) \wedge \text{Male}(x) \rightarrow \text{Father}(x, y)$$

Make the specification as precise as possible; e.g., in the case of Father , also add:

$$\begin{aligned} \text{Father}(x, y) &\rightarrow \text{Parent}(x, y) \\ \text{Father}(x, y) &\rightarrow \text{Male}(x) \end{aligned}$$

Try to be more succinct by reusing the specified predicates; e.g., use Sibling in the specification of Brother and Sister . Explain the nature of the problem in any case where expressivity limitations of Datalog make exact specification impossible.

Problem 4. Consider the Datalog knowledge base $\mathcal{K} = \langle \mathcal{R}, \mathcal{F} \rangle$ where \mathcal{F} contains the following facts:

$$\mathcal{F} = \{\text{Father}(\text{john}, \text{mary}), \text{Mother}(\text{mary}, \text{peter}), \text{Father}(\text{john}, \text{david})\}$$

and \mathcal{R} contains the following rules:

$$\text{Parent}(x, y) \wedge \text{Parent}(y, z) \rightarrow \text{GrandParent}(x, z) \quad (1)$$

$$\text{Parent}(x, y) \wedge \text{Parent}(x, z) \rightarrow \text{Sibling}(y, z) \quad (2)$$

$$\text{Father}(x, y) \rightarrow \text{Parent}(x, y) \quad (3)$$

$$\text{Mother}(x, y) \rightarrow \text{Parent}(x, y) \quad (4)$$

Do the following:

1. Using backward-chaining show that John is a grand parent of Peter.
2. Using forward-chaining show that Mary and David are siblings.