Conditionals in Human Reasoning under the Weak Completion Semantics

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Suppression Task, First Part

Facts	Conditional	Alternative Argument	Additional Argument
	If she has an essay to finish, then she will stay late in the library	If she has a textbook to read, then she will stay late in the library	If the library stays open, then she will stay late in the library
She has an essay to finish	She will study late in the library (96% L)	She will study late in the library (96% L)	She will study late in the library (38% L)
She does not have an essay to finish	She will not study late in the library (46% $\neg L$)	She will not study late in the library (4% $\neg L$)	She will not study late in the library (63% $\neg L$)

Additional Arguments lead to the suppression of previously drawn conclusions.
Alternative Arguments lead to the suppression of previously drawn conclusions.

A Classical Logic Approach to the Suppression Task

If she has an essay to finish	then she will stay late in the library	$l \leftarrow e$
If she has a textbook to read	then she will stay late in the library	$l \leftarrow t$
If the library stays open	then she will stay late in the library	$l \leftarrow o$

Clauses	Facts	Classical Logic	Exp. Findings	
$l \leftarrow e$	e	$\models \; l$	96% L	Modus Ponens
$l \leftarrow e \;\; l \leftarrow t$	e	$\models \; l$	96% L	Modus Ponens
$l \leftarrow e \;\; l \leftarrow o$	e	$\models \; l$	38% L	Modus Ponens
$l \leftarrow e$	$\neg e$	$\not\models \neg l$	46% $\neg L$	Denial of the Antecedent
$l \leftarrow e \;\; l \leftarrow t$	$\neg e$	$\not\models \neg l$	4% $\neg L$	Denial of the Antecedent
$l \leftarrow e \;\; l \leftarrow o$	$\neg e$	$\not\models \neg l$	63% $\neg L$	Denial of the Antecedent

Classical logic does not adequately represent the suppression task.

Computational Logic Approach

Hölldobler and Kencana Ramli [2009b] propose using:

- Logic programs
- under weak completion semantics
- based on three-valued Łukasiewicz logic.

We will show that this approach seems to adequately represent the results of Byrne's suppression task and Wason's Selection Task.

Byrne's Suppression Task

Part I

Logic Programs

A logic program $\mathcal{P}$ is a finite set of clauses of the form

$$A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$$

A is an atom called head of the clause, and $A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$ is a conjunction of (negated) atoms called body of the clause.

Representation of the Suppression Task (Stenning and van Lambalgen [2008])

$\mathcal{P}$	Clauses				Facts
$\mathcal{P}_e$	$l \leftarrow e \wedge \neg ab_1$		$ab_1 \leftarrow \bot$		$e \leftarrow \top$
$\mathcal{P}_{e+Add}$	$l \leftarrow e \wedge \neg ab_1$	$l \leftarrow o \wedge \neg ab_3$	$ab_1 \leftarrow \neg o$	$ab_3 \leftarrow \neg e$	$e \leftarrow \top$
$\mathcal{P}_{\neg e+Alt}$	$l \leftarrow e \wedge \neg ab_1$	$l \leftarrow t \wedge \neg ab_2$	$ab_1 \leftarrow \bot$	$ab_2 \leftarrow \bot$	$e \leftarrow \bot$

- $A \leftarrow \top$ is a positive fact and $A \leftarrow \bot$ is a negative fact.
- The abnormality predicates are by default mapped to $\bot$.
- They allow to express the dependencies between clauses.

The Weak Completion of a Logic Program

$wc\mathcal{P}$	Clauses			Facts
$wc\mathcal{P}_e$	$l \leftrightarrow e \wedge \neg ab_1$	$ab_1 \leftrightarrow \bot$		$e \leftrightarrow \top$
$wc\mathcal{P}_{e+Add}$	$l \leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_3)$	$ab_1 \leftrightarrow \neg o$	$ab_3 \leftrightarrow \neg e$	$e \leftrightarrow \top$
$wc\mathcal{P}_{\neg e+Alt}$	$l \leftrightarrow (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2)$	$ab_1 \leftrightarrow \bot$	$ab_2 \leftrightarrow \bot$	$e \leftrightarrow \bot$

1. All clauses with the same head $A \leftarrow body_1,\ A \leftarrow body_2,\ \ldots$ are replaced by $A \leftarrow body_1 \vee body_2 \vee \ldots$.
2. All occurrences of $\leftarrow$ are replaced by $\leftrightarrow$.

Computing Least Models

$\mathcal{P}_{e+Add} = \{l \leftarrow e \wedge \neg ab_1, l \leftarrow o \wedge \neg ab_3, ab_1 \leftarrow \neg o, ab_3 \leftarrow \neg e, e \leftarrow \top\}$

The least fixed point of $\Phi_{\mathcal{P}}$ ($\text{lfp}\,\Phi_{\mathcal{P}}$) (Stenning and van Lambalgen [2008]) is identical to the least model of the weak completion of $\mathcal{P}$ ($\text{lm}_{\text{Ł}}\text{wc}\,\mathcal{P}$).

Consider an interpretation $I = \langle I^\top, I^\bot \rangle$, starting with $I_0 = \langle \emptyset, \emptyset \rangle$:

$$
\begin{array}{lllllllll}
I_1 & = & \Phi_{\mathcal{P}_{e+Add}}(I_0) & = & \langle \{e\}, \emptyset \rangle & & & & \\
I_2 & = & \Phi_{\mathcal{P}_{e+Add}}(I_1) & = & \langle \{e\}, \{ab_3\} \rangle & = & \Phi_{\mathcal{P}_{e+Add}}(I_2) & \Leftarrow & \text{lm}_{\text{Ł}}\text{wc}\,(\mathcal{P}_{e+Add})
\end{array}
$$

- $A \in I^\top$ if there exists $A \leftarrow body \in \mathcal{P}$ with $I(body) = \top$
- $A \in I^\bot$ if there exists $A \leftarrow body \in \mathcal{P}$ and for all $A \leftarrow body : I(body) = \bot$

The Results of the First Part of the Suppression Task

$\mathcal{P}$	Weak Completion Semantics	Well-founded Semantics	Byrne
$\mathcal{P}_e$	$\models_{Ł}^{lmwc}\ l$	$\models^{wfm}\ l$	96% L
$\mathcal{P}_{e+Alt}$	$\models_{Ł}^{lmwc}\ l$	$\models^{wfm}\ l$	96% L
$\mathcal{P}_{e+Add}$	$\not\models_{Ł}^{lmwc}\ l$	$\models^{wfm}\ \neg l$	38% L
$\mathcal{P}_{\neg e}$	$\models_{Ł}^{lmwc}\ \neg l$	$\models^{wfm}\ \neg l$	46% $\neg L$
$\mathcal{P}_{\neg e+Alt}$	$\not\models_{Ł}^{lmwc}\ \neg l$	$\models^{wfm}\ \neg l$	4% $\neg L$
$\mathcal{P}_{\neg e+Add}$	$\models_{Ł}^{lmwc}\ \neg l$	$\models^{wfm}\ \neg l$	63% $\neg L$

Byrne's Suppression Task

Part II

Suppression Task, Second Part

Facts	Conditional	Alternative Argument	Additional Argument
	If she has an essay to finish, then she will stay late in the library	If she has a textbook to read, then she will stay late in the library	If the library stays open, then she will stay late in the library
She stays late in the library	She has an essay to finish (53% E)	She has an essay to finish (16% E)	She has an essay to finish (55% E)
She does not stay late in the library	She does not have an essay to write (69% $\neg E$)	She does not have an essay to write (69% $\neg E$)	She does not have an essay to write (44% $\neg E$)

Participants abductively explain why she stays late in the library. Participants abductively explain why she does not stay late in the library.

A Classical Logic Approach to the Second Part of the Suppression Task

If she has an essay to finish	then she will stay late in the library	$l \leftarrow e$
If she has a textbook to read	then she will stay late in the library	$l \leftarrow t$
If the library stays open	then she will stay late in the library	$l \leftarrow o$

Clauses	Facts	Classical Logic	Exp. Findings	
$l \leftarrow e$	l	$\not\models e$	53% E	Affirmation of the Consequent
$l \leftarrow e \quad l \leftarrow t$	l	$\not\models e$	16% E	Affirmation of the Consequent
$l \leftarrow e \quad l \leftarrow o$	l	$\not\models e$	55% E	Affirmation of the Consequent
$l \leftarrow e$	$\neg l$	$\models \neg e$	69% $\neg E$	Modus Tollens
$l \leftarrow e \quad l \leftarrow t$	$\neg l$	$\models \neg e$	69% $\neg E$	Modus Tollens
$l \leftarrow e \quad l \leftarrow o$	$\neg l$	$\models \neg e$	44% $\neg E$	Modus Tollens

Classical logic does not adequately represent the suppression task.

Abduction (Kakas et al. [1993])

Given an abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{Ł}^{lmwc} \rangle$ where

- set of abducibles $\mathcal{A}$ contains all positive and negative facts of each $A \in \text{undef}(\mathcal{P})$,
- $\mathcal{E}$ is an explanation and a consistent subset of $\mathcal{A}$,
- logical consequence relation $\models_{Ł}^{lmwc}$, where $\mathcal{P} \models_{Ł}^{lmwc} F$ iff $\text{lm}_{Ł}\text{wc}\,\mathcal{P}(F) = \top$, and
- $\mathcal{O}$ is an observation which is a set of (at least one) literals.

$\mathcal{O}$ is **explained by** $\mathcal{E}$ **given** $\mathcal{P}$ iff $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} O$, where $\mathcal{P} \not\models_{Ł}^{lmwc} \mathcal{O}$

$\mathcal{O}$ is **explained given** $\mathcal{P}$ iff there exists an $\mathcal{E}$ such that $\mathcal{O}$ is explained by $\mathcal{E}$ given $\mathcal{P}$

F **follows skeptically from** $\mathcal{P}$ **and** $\mathcal{O}$ iff $\mathcal{O}$ can be explained given $\mathcal{P}$ and for all minimal (or otherwise preferred) explanations $\mathcal{E}$ we find that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} \mathcal{O}$.

F **follows credulously from** $\mathcal{P}$ **and** $\mathcal{O}$ iff there exists a minimal (or otherwise preferred) explanation $\mathcal{E}$ such that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} \mathcal{O}$.

Abduction within the Second Part of the Suppression Task

	Conditionals	$\mathcal{O}$	Minimal $\mathcal{E}$s	Byrne	
$\mathcal{P}_{l}$	$=\{l \leftarrow e \wedge \neg ab_1, \quad ab_1 \leftarrow \bot\}$	$\{l\}$	$\{e \leftarrow \top\}$	53%	E
$\mathcal{P}_{l+Alt}$	$=\{l \leftarrow e \wedge \neg ab_1, l \leftarrow t \wedge \neg ab_2, ab_1 \leftarrow \bot, \ ab_2 \leftarrow \bot\}$	$\{l\}$	$\{e \leftarrow \top\}, \{t \leftarrow \top\}$	16%	E
$\mathcal{P}_{l+Add}$	$=\{l \leftarrow e \wedge \neg ab_1, l \leftarrow o \wedge \neg ab_3, ab_1 \leftarrow \neg o, ab_3 \leftarrow \neg e\}$	$\{l\}$	$\{e \leftarrow \top, \ o \leftarrow \top\}$	55%	E
$\mathcal{P}_{\neg l}$	$=\{l \leftarrow e \wedge \neg ab_1, \quad ab_1 \leftarrow \bot\}$	$\{\neg l\}$	$\{e \leftarrow \bot\}$	69%	$\neg E$
$\mathcal{P}_{\neg l+Alt}$	$=\{l \leftarrow e \wedge \neg ab_1, l \leftarrow t \wedge \neg ab_2, ab_1 \leftarrow \bot, \ ab_2 \leftarrow \bot\}$	$\{\neg l\}$	$\{e \leftarrow \bot, \ t \leftarrow \bot\}$	59%	$\neg E$
$\mathcal{P}_{\neg l+Add}$	$=\{l \leftarrow e \wedge \neg ab_1, l \leftarrow o \wedge \neg ab_3, ab_1 \leftarrow \neg o \ ab_3 \leftarrow e\}$	$\{\neg l\}$	$\{e \leftarrow \bot\}, \{o \leftarrow \bot\}$	44%	$\neg E$

The Results of the Second Part of the Suppression Task

$\mathcal{P}$	$\mathcal{O}$	$\mathcal{E}$	Weak Completion Semantics	Well-founded Semantics	Byrne
$\mathcal{P}_{l}$	l	$e \leftarrow \top$	$\models_{Ł}^{\text{lmwc}}\ e$	$\models^{\text{wfm}}\ e$	53% E
$\mathcal{P}_{l+Alt}$	l	$e \leftarrow \top$	$\models_{Ł}^{\text{lmwc}}\ e$	$\models^{\text{wfm}}\ e$	16% E
		$t \leftarrow \top$	$\not\models_{Ł}^{\text{lmwc}}\ e$	$\models^{\text{wfm}}\ \neg e$	
$\mathcal{P}_{l+Add}$	l	$e \leftarrow \top, o \leftarrow \top$	$\models_{Ł}^{\text{lmwc}}\ e$	$\models^{\text{wfm}}\ e$	55% E
$\mathcal{P}_{\neg l}$	$\neg l$	$e \leftarrow \bot$	$\models_{Ł}^{\text{lmwc}}\ \neg e$	$\models^{\text{wfm}}\ \neg e$	69% $\neg E$
$\mathcal{P}_{\neg l+Alt}$	$\neg l$	$e \leftarrow \bot, t \leftarrow \bot$	$\models_{Ł}^{\text{lmwc}}\ \neg e$	$\models^{\text{wfm}}\ \neg e$	69% $\neg E$
$\mathcal{P}_{\neg l+Add}$	$\neg l$	$e \leftarrow \bot$	$\models_{Ł}^{\text{lmwc}}\ \neg e$	$\models^{\text{wfm}}\ \neg e$	44% $\neg E$
		$o \leftarrow \bot$	$\not\models_{Ł}^{\text{lmwc}}\ \neg e$	$\models^{\text{wfm}}\ \neg e$	

In the two cases where WCS and WFM differ, it is not clear, which one adequately models the participants' conclusions.

Wason's Selection Task

The Wason Selection Task: Abstract Case (Wason 1968)

Consider four cards where each of them has a letter on one side and a number on the other side. Given the conditional

If there is a D on one side of the card, then there is a 3 on the other side.

Which cards must be turned to prove that the conditional holds?

	D	F	3	7
Experimental Results	89%	16%	62%	25%

The Wason Selection Task: Social Case (Griggs and Cox 1982)

Consider four cards, where on one side there is the person's age and on the other side of the card what the person is drinking. Given the conditional

If a person is drinking beer, then the person must be over 19 years of age.

Which cards must be turned to prove that the conditional holds?

	beer	coke	22yrs	16yrs
Experimental Results	95%	0.025%	0.025%	80%

Modeling the

Social Case

of the Selection Task

Social Case of the Selection Task

According to Kowalski the social case is understood as a social constraint.

The conditional is encoded as a goal $\mathcal{G} = \{o \leftarrow b \wedge \neg ab_1\}$,
where o means 'a person older than 19 years', b means 'drinking beer'
and ab_1 is an abnormality predicate.

$$wc(\mathcal{P}_{beer}) = \{ab_1 \leftrightarrow \bot, b \leftrightarrow \top\}$$

case	program $\mathcal{P}$	$\mathrm{lm}_{Ł}\,\mathrm{wc}\,\mathcal{P}$			Griggs & Cox
beer	$\{ab_1 \leftarrow \bot, b \leftarrow \top\}$	$\langle\{b\}, \{ab_1\}\rangle$	$\not\models_{Ł}$	$o \leftarrow b \wedge \neg ab_1$	95%
coke	$\{ab_1 \leftarrow \bot, b \leftarrow \bot\}$	$\langle\emptyset, \{b, ab_1\}\rangle$	$\models_{Ł}$	$o \leftarrow b \wedge \neg ab_1$	0.025%
22yrs	$\{ab_1 \leftarrow \bot, o \leftarrow \top\}$	$\langle\{o\}, \{ab_1\}\rangle$	$\models_{Ł}$	$o \leftarrow b \wedge \neg ab_1$	0.025%
16yrs	$\{ab_1 \leftarrow \bot, o \leftarrow \bot\}$	$\langle\emptyset, \{o, ab_1\}\rangle$	$\not\models_{Ł}$	$o \leftarrow b \wedge \neg ab_1$	80%

Modeling the

Abstract Case

of the Selection Task

Abstract Case of the Selection Task

According to Kowalski, the abstract case is understood as a belief.

Consider $\mathcal{P} = \{3 \leftarrow D \wedge \neg ab_2, ab_2 \leftarrow \bot\}$ with $\text{lm}_{Ł}\text{wc}\,\mathcal{P} = \langle \emptyset, \{ab_2\} \rangle$.
$\langle \emptyset, \{ab_2\} \rangle$ does not explain any letter on a card.
The set of abducibles is $\{D \leftarrow \top,\ D \leftarrow \bot,\ F \leftarrow \top,\ F \leftarrow \bot,\ 7 \leftarrow \top,\ 7 \leftarrow \bot\}$.

$$\text{wc}(\mathcal{P} \cup \mathcal{E}_D) = \{D \leftrightarrow 3 \wedge \neg ab_2, D \leftrightarrow \top\}$$

$\mathcal{O}$	$\mathcal{E}$	$\text{lm}_{Ł}\text{wc}\,(\mathcal{P} \cup \mathcal{E})$			Wason
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, \{ab_2\} \rangle$	$\rightsquigarrow$	turn	89%
F	$\{F \leftarrow \top\}$	$\langle \{F\}, \{ab_2\} \rangle$	$\rightsquigarrow$	no turn	16%
3	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, \{ab_2\} \rangle$	$\rightsquigarrow$	turn	62%
7	$\{7 \leftarrow \top\}$	$\langle \{7\}, \{ab_2\} \rangle$	$\rightsquigarrow$	no turn	25%

Conclusion

1. Stenning and van Lambalgen (2008) formalize Byrne's Suppression Task.
2. Kowalski (2011) models Wason's Selection Task.
3. Hölldobler and Kencana Ramli (2009a) found technical mistakes done by Stenning and van Lambalgen and propose to model human reasoning by

- logic programs
- under weak completion semantics
- based on the three-valued Łukasiewicz (1920) logic.

The weak completion semantics seems to adequately model both, Byrne's Suppression Task and Wason's Selection Task!

Can we model other reasoning processes such as syllogistic reasoning?

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