TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

Dr. Hannes Strass M.A. Jonas Karge

Knowledge Representation and Reasoning

Winter 2024 Term

Exercises 11

03–07/02/2025

Problem 1.
Compute the least fixpoint of the operator ${}_{P_1}T$ for the definite logic program P_1 defined as follows:

$P_1 =$

$$\begin{array}{rl@{\qquad\qquad}rl} p \leftarrow & q, r & p \leftarrow & s, t \\ w \leftarrow & s, r & z \leftarrow & p, w \\ s \leftarrow & p, q & v \leftarrow & w \\ r \leftarrow & t, m & q \leftarrow & \\ t \leftarrow & & m \leftarrow & \end{array}$$

Problem 2.
Approximation Fixpoint Theory can be used to define the semantics of argumentation frameworks. For an AF $F = (A, R)$ we thus consider the complete lattice $(L, \subseteq)$ with $L = 2^A$ and its associated bilattice $(L^2, \leq_i)$.

The stable extension semantics is characterised by the operator ${}_FU$ defined as follows:

Definition. For each AF $F = (A, R)$, the operator ${}_FU : A \to A$ yields – for a given set $S \subseteq A$ – a new set

$${}_FU(S) = \{a \in A \mid b \in S \text{ implies } (b, a) \notin R\}$$

Intuitively, the operator returns the set of all arguments that are unattacked (hence the U) by the input set.

An approximator of ${}_FU$ is given by ${}_F\mathcal{U}$ defined as follows:

Definition. For each AF $F = (A, R)$, the operator ${}_F\mathcal{U} : 2^A \times 2^A \to 2^A \times 2^A$ yields – for a given pair $(P, S) \in 2^A \times 2^A$ – a new pair

$${}_F\mathcal{U}(P, S) = ({}_FU(S), {}_FU(P))$$

Now, consider the two AFs: $F_1 = (A_1, R_1)$ with $A_1 = \{a, b\}$ and $R_1 = \{a, b, (b, a)\}$ and $F_2 = (A_2, R_2)$ with $A_2 = \{a, b, c\}$ and $R_2 = \{(a, b), (b, c), (c, a)\}$.

Do the following:

(i) For both frameworks F_i, $i = 1, 2$, illustrate the corresponding complete lattice $(2^{A_i}, \leq_i))$ and the mappings of ${}_{F_i}U$.

(ii) Show that ${}_FU$ characterises stable extension semantics. More precisely: Show that for every AF F, every set $S \subseteq A$ is a stable extension of F iff ${}_FU(S) = S$.

(iii) Argue whether or not ${}_FU$ is $\subseteq$-monotone for every AF F.

(iv) Show that ${}_F\mathcal{U}$ is an approximator for ${}_FU$.

Problem 3.
Consider the normal logic program P_2:

$P_2 =$

$$\begin{array}{ll} p \leftarrow p, q & q \leftarrow p \\ r \leftarrow \sim s & s \leftarrow \sim q \end{array}$$

Recall the definition of the approximator ${}_{P_2}\mathcal{T}$ from the lecture. Compute the least fixpoint of ${}_{P_2}\mathcal{T}$ for the normal logic program P_2.