

Exercise Sheet 9: Advanced L^AT_EX
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Exercise 9.1. Typeset the following paragraph.

Lemma 8.2 (Yoneda) *Let \mathbf{C} be locally small. For any object $C \in \mathbf{C}$ and functor $F \in \mathbf{Sets}^{\mathbf{C}^{op}}$ there is an isomorphism*

$$\text{Hom}(yC, F) \cong FC$$

which, moreover, is natural in both F and C .

Here:

- (1) *the Hom is $\text{Hom}_{\mathbf{Sets}^{\mathbf{C}^{op}}}$,*
- (2) *naturality in F means that, given any $\vartheta : F \rightarrow G$, the following diagram commutes:*

$$\begin{array}{ccc} \text{Hom}(yC, F) & \xrightarrow{\cong} & FC \\ \text{Hom}(yC, \vartheta) \downarrow & & \downarrow \vartheta_C \\ \text{Hom}(yC, G) & \xrightarrow{\cong} & GC \end{array}$$

- (3) *naturality in C means that, given any $h : C \rightarrow D$, the following diagram commutes:*

$$\begin{array}{ccc} \text{Hom}(yC, F) & \xrightarrow{\cong} & FC \\ \text{Hom}(yh, F) \uparrow & & \uparrow Fh \\ \text{Hom}(yD, F) & \xrightarrow{\cong} & FD \end{array}$$

(Awodey, Steve. 2006. Category Theory. Oxford University Press. p. 162)

Hint: You can use `tikz`¹ with the `tikz-cd` library to typeset the *commutative diagrams*, but several other packages are also available.

Exercise 9.2. Write a paper proving the *binomial theorem*:

Let $n \in \mathbb{N}$. Then

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

You may use any sources that you need, provided that you properly cite them. Make sure your paper includes an introduction, a conclusion, and all necessary preliminaries. Try to make your paper as easy to read as possible.

¹<https://github.com/pgf-tikz/pgf>