Contextual Reasoning

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Jerry is a Bird. Can Jerry fly?

The Φ operator applied to interpretation I and program $\mathcal{P}$, denoted by $\Phi_{\mathcal{P}}(I)$, is the interpretation $J = \langle J^{\top}, J^{\bot} \rangle$:

$$
\begin{array}{lcl}
J^{\top} & = & \{A \mid \text{there exists a clause } A \leftarrow \mathit{body} \in g\mathcal{P} \text{ and } I(\mathit{body}) = \top\}, \\
J^{\bot} & = & \{A \mid \text{there exists a clause } A \leftarrow \mathit{body} \text{ and for all } A \leftarrow \mathit{body} \in g\mathcal{P} \text{ we find that } I(\mathit{body}) = \bot\}.
\end{array}
$$

$$
\begin{array}{lll}
\mathcal{P} & = \{ & \mathit{fly}(X) \leftarrow \mathit{bird}(X) \wedge \neg \mathit{ab}_1(X), \quad \mathit{bird}(\mathit{jerry}) \leftarrow \top, \\
& & \mathit{ab}_1(X) \leftarrow \mathit{kiwi}(X), \\
& & \mathit{ab}_1(X) \leftarrow \mathit{penguin}(X) \ \}
\end{array}
$$

$$
\begin{array}{lll}
g\mathcal{P} & = \{ & \mathit{fly}(\mathit{jerry}) \leftarrow \mathit{bird}(\mathit{jerry}) \wedge \neg \mathit{ab}_1(\mathit{jerry}), \quad \mathit{bird}(\mathit{jerry}) \leftarrow \top, \\
& & \mathit{ab}_1(\mathit{jerry}) \leftarrow \mathit{kiwi}(\mathit{jerry}), \\
& & \mathit{ab}_1(\mathit{jerry}) \leftarrow \mathit{penguin}(\mathit{jerry}) \ \}
\end{array}
$$

Starting with the empty interpretation $I_0 = \langle \emptyset, \emptyset \rangle$,

$$
\begin{array}{lclcl}
\Phi_{\mathcal{P}}(I_0) & = & \langle \{\mathit{bird}(\mathit{jerry})\}, \emptyset \rangle & = & I_1 \\
\Phi_{\mathcal{P}}(I_1) & = & \langle \{\mathit{bird}(\mathit{jerry})\}, \emptyset \rangle & = & \mathrm{lfp}\ (\Phi_{\mathcal{P}}(I_0))
\end{array}
$$

How to deal with Exception Cases?

- We want to avoid explicitly stating that all exceptions are false
- We don't think that humans apply the closed world assumption in reality
- We assume that they don't consciously consider all exceptions*, but instead
- if they are not for some reason aware of the exceptions, they simply ignore them

- An approach that intends to adequately model human reasoning, should,
  - instead of applying the closed world assumption,
  - leave the truth values of these exception cases unknown and
  - find a mechanism that ignores them

At the moment, we cannot express this idea syntactically
under the Weak Completion Semantics!

* Currently, we know of at least 40 species of birds that cannot fly.

Contextual Programs

An contextual (datalog) program $\mathcal{P}$ is a finite set of clauses:

$$
\begin{aligned}
A &\leftarrow L_1 \wedge \ldots \wedge L_m \wedge \text{ctxt}(L_{m+1}) \wedge \ldots \wedge \text{ctxt}(L_{m+p}) && (1)\\
A &\leftarrow \top && (2)\\
A &\leftarrow \bot && (3)
\end{aligned}
$$

- A is an atom and the L_i with $0 \leq i \leq m+p$ are literals
- A is called head and $L_1 \wedge \ldots \wedge L_n$ as well as $\top$ and $\bot$ are called bodies of the corresponding clauses.
- In case $m = p = 0$, then the clause is a fact and written as (2)
- A clause of the form (2) and (3) is a fact and assumption, respectively
- $\neg A$ is assumed if the only clause in $\mathcal{P}$ of which it is the head of, is an assumption
- ctxt is a new truth functional operator
- $g\mathcal{P}$ is ground $\mathcal{P}$ wrt the constants occurring in $\mathcal{P}$
- A is undefined in $\mathcal{P}$ if it is not the head of any clause in $\mathcal{P}$

Three-valued Łukasiewicz (1920) Semantics extended by ctxt

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

A	$\neg A$	ctxt(A)	ctxt($\neg A$)
$\top$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\top$
U	U	$\bot$	$\bot$

- I is a model of $\mathcal{P}$ iff for all $A \leftarrow body \in g\mathcal{P}$ it holds that $I(A \leftarrow body) = \top$
- I is a model of wc $\mathcal{P}$ iff for all $A \leftrightarrow body_1 \vee body_2 \cdots \vee body_n \in$ wc $g\mathcal{P}$ it holds that $I(A \leftrightarrow body_1 \vee body_2 \cdots \vee body_n) = \top$

$$\mathcal{P}_1 = \{p \leftarrow \text{ctxt}(q)\} \qquad \mathcal{P}_2 = \{p \leftarrow q\}$$
$$\text{wc}\,\mathcal{P}_1 = \{p \leftrightarrow \text{ctxt}(q)\} \qquad \text{wc}\,\mathcal{P}_2 = \{p \leftrightarrow q\}$$

Which of the interpretations is a model of the programs and their weak completion?

- $I_1 = \langle \{p, q\}, \emptyset \rangle \Rightarrow$ model of $\mathcal{P}_1, \mathcal{P}_2$, wc $\mathcal{P}_1$ and wc $\mathcal{P}_2$
- $I_2 = \langle \emptyset, \emptyset \rangle \Rightarrow$ model of $\mathcal{P}_1, \mathcal{P}_2$ and wc $\mathcal{P}_2$
- $I_3 = \langle \emptyset, \{p\} \rangle \Rightarrow$ model of $\mathcal{P}_1$ and wc $\mathcal{P}_1$

Characteristics of ctxt

Φ applied to I and $\mathcal{P}$, denoted by $\Phi_{\mathcal{P}}(I)$, is the interpretation $J = \langle J^{\top}, J^{\bot} \rangle$:

$$J^{\top} = \{A \mid \text{there exists a clause } A \leftarrow body \in g\mathcal{P} \text{ and } I(body) = \top\},$$
$$J^{\bot} = \{A \mid \text{there exists a clause } A \leftarrow body \text{ and for all } A \leftarrow body \in g\mathcal{P} \text{ we find that } I(body) = \bot\}.$$

$$\mathcal{P}_1 = \{p \leftarrow q\} \qquad \mathcal{P}_2 = \{p \leftarrow \mathrm{ctxt}(q)\}$$

Starting with the empty interpretation $I_0 = \langle \emptyset, \emptyset \rangle$,

$$\mathrm{lfp}\ (\Phi_{\mathcal{P}_1}) = \langle \emptyset, \emptyset \rangle \quad \text{and} \quad \mathrm{lfp}\ (\Phi_{\mathcal{P}_2}) = \langle \emptyset, \{p\} \rangle$$

A	$\neg A$	$\mathrm{ctxt}(A)$	$\mathrm{ctxt}(\neg A)$
$\top$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\top$
U	U	$\bot$	$\bot$

Nothing is known about q. Therefore $\mathrm{ctxt}(q)$ is false.
We derive that p is false in $\mathrm{lfp}\ (\Phi_{\mathcal{P}_2})$.

Contextual Abduction

A contextual abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{wcs} \rangle$ consists of

- a contextual program $\mathcal{P}$
- a set of abducibles $\mathcal{A} \subseteq \mathcal{A}_\mathcal{P}$, where

$$\begin{array}{lcl} \mathcal{A}_\mathcal{P} & = & \{A \leftarrow \top \mid A \text{ is undefined in } \mathcal{P}\} \\ & \cup & \{A \leftarrow \bot \mid A \text{ is undefined in } \mathcal{P}\} \\ & \cup & \{A \leftarrow \top \mid \neg A \text{ is assumed in } \mathcal{P}\}, \end{array}$$

- the entailment relation $\models_{wcs}$, where $\mathcal{P} \models_{wcs} F$ iff $\text{lfp}\,(\Phi_\mathcal{P})(F) = \top$

Let $\langle \mathcal{P}, \mathcal{A}, \models_{wcs} \rangle$ be a contextual abductive framework, $\mathcal{E} \subseteq \mathcal{A}$ and the observation $\mathcal{O}$ is a set of ground literals.

$\mathcal{O}$ is *contextually explained by* $\mathcal{E}$ *given* $\mathcal{P}$ iff
$\mathcal{P} \cup \mathcal{E} \models_{wcs} L$ for all $L \in \mathcal{O}$ and for all $A \leftarrow \top \in \mathcal{E}$ and $A \leftarrow \bot \in \mathcal{E}$
there exists an $L \in \mathcal{O}$, such that L restrictly depends on A.

We assume explanations to be minimal.

The restrictly depends on Relation

Let $\langle \mathcal{P}, \mathcal{A}, \models_{wcs} \rangle$ be a contextual abductive framework, $\mathcal{E} \subseteq \mathcal{A}$ and the observation $\mathcal{O}$ is a set of ground literals.

> $\mathcal{O}$ is *contextually explained by* $\mathcal{E}$ *given* $\mathcal{P}$ iff
> $\mathcal{P} \cup \mathcal{E} \models_{wcs} L$ for all $L \in \mathcal{O}$, and for all $A \leftarrow \top \in \mathcal{E}$ and $A \leftarrow \bot \in \mathcal{E}$ there exists an $L \in \mathcal{O}$, such that L restrictly depends on A.

Given a clause $A \leftarrow L_1 \wedge \ldots \wedge L_m \wedge \text{ctxt}(L_{m+1}) \wedge \ldots \wedge \text{ctxt}(L_{m+p})$ for all $1 \leq i \leq m$, A restrictly depends on L_i. The restrictly depends on relation is transitive.
If A restrictly depends on L_i, then $\neg A$ restrictly depends on L_i. Furthermore, if $L_i = B$, then A restrictly depends on $\neg B$ and if $L_i = \neg B$, then A restrictly depends on B.

$$\mathcal{P} \quad = \quad \{ \quad p \leftarrow r, \quad p \leftarrow \text{ctxt}(q) \quad \}$$

- p restrictly depends on r, $\neg p$ restrictly depends on $\neg r$, ...
- p does not restrictly depend on q, neither restrictly depends on $\text{ctxt}(q)$.

Jerry is a bird. Can Jerry fly?

$$\begin{aligned} \mathcal{P} \quad = \{ \quad & \mathit{fly}(X) \leftarrow \mathit{bird}(X) \wedge \neg ab_1(X), \quad \mathit{bird}(\mathit{jerry}) \leftarrow \top, \\ & ab_1(X) \leftarrow \mathit{kiwi}(X), \\ & ab_1(X) \leftarrow \mathit{penguin}(X) \ \} \end{aligned}$$

Reconsider

$$\mathcal{O} \quad = \quad \{\mathit{fly}(\mathit{jerry})\}$$

$$\begin{aligned} \mathcal{P}' \quad = \{ \quad & \mathit{fly}(X) \leftarrow \mathit{bird}(X) \wedge \neg ab_1(X), \quad \mathit{bird}(\mathit{jerry}) \leftarrow \top, \\ & ab_1(X) \leftarrow \mathrm{ctxt}(\mathit{kiwi}(X)), \\ & ab_1(X) \leftarrow \mathrm{ctxt}(\mathit{penguin}(X)) \ \} \end{aligned}$$

$$\begin{aligned} g\mathcal{P}' \quad = \{ \quad & \mathit{fly}(\mathit{jerry}) \ \leftarrow \ \mathit{bird}(\mathit{jerry}) \wedge \neg ab_1(\mathit{jerry}), \quad \mathit{bird}(\mathit{jerry}) \leftarrow \top, \\ & ab_1(\mathit{jerry}) \ \leftarrow \ \mathrm{ctxt}(\mathit{kiwi}(\mathit{jerry})), \\ & ab_1(\mathit{jerry}) \ \leftarrow \ \mathrm{ctxt}(\mathit{penguin}(\mathit{jerry})) \ \} \end{aligned}$$

Jerry is a bird. Jerry can fly!

Starting with the empty interpretation $I_0 = \langle \emptyset, \emptyset \rangle$,

$$\begin{array}{lclcl} \Phi_{\mathcal{P}'}(I_0) & = & \langle \{bird(jerry)\}, \{ab_1(jerry)\} \rangle & = & I_1 \\ \Phi_{\mathcal{P}'}(I_1) & = & \langle \{fly(jerry), bird(jerry)\}, \{ab_1(jerry)\} \rangle & = & \text{lfp } (\Phi_{\mathcal{P}}(I_0)) \end{array}$$

A	$\neg A$	ctxt(A)	ctxt($\neg A$)
$\top$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\top$
U	U	$\bot$	$\bot$

$\mathcal{O} = \{fly(jerry)\}$ follows immediately from $\mathcal{P}'$ and $\mathcal{E} = \emptyset$.

We do not explicitly state that Jerry is not a *kiwi* nor a *penguin*!

Why does Tweety not fly? (1)

$$\mathcal{P}'' = \{ \begin{array}{rcl} \mathit{fly}(X) & \leftarrow & \mathit{bird}(X) \wedge \neg \mathit{ab}_1(X), \\ \mathit{ab}_1(X) & \leftarrow & \mathrm{ctxt}(\mathit{kiwi}(X)) \\ \mathit{ab}_1(X) & \leftarrow & \mathrm{ctxt}(\mathit{penguin}(X)), \\ \mathit{bird}(\mathit{tweety}) & \leftarrow & \top, \\ \mathit{kiwi}(X) & \leftarrow & \mathit{featherslikeHair}(X), \\ \mathit{penguin}(X) & \leftarrow & \mathit{blackAndWhite}(X) \; \} \end{array}$$

$$\mathcal{O} = \{ \; \neg \mathit{fly}(\mathit{tweety}), \quad \mathit{featherslikeHair}(\mathit{tweety}) \; \}$$

$$g\mathcal{P}'' = \{ \begin{array}{rcl} \mathit{fly}(\mathit{tweety}) & \leftarrow & \mathit{bird}(\mathit{tweety}) \wedge \neg \mathit{ab}_1(\mathit{tweety}), \\ \mathit{ab}_1(\mathit{tweety}) & \leftarrow & \mathrm{ctxt}(\mathit{kiwi}(\mathit{tweety})), \\ \mathit{ab}_1(\mathit{tweety}) & \leftarrow & \mathrm{ctxt}(\mathit{penguin}(\mathit{tweety})), \\ \mathit{bird}(\mathit{tweety}) & \leftarrow & \top, \\ \mathit{kiwi}(\mathit{tweety}) & \leftarrow & \mathit{featherslikeHair}(\mathit{tweety}), \\ \mathit{penguin}(\mathit{tweety}) & \leftarrow & \mathit{blackAndWhite}(\mathit{tweety}) \; \} \end{array}$$

$$\mathcal{O} = \{ \; \neg \mathit{fly}(\mathit{tweety}), \quad \mathit{featherslikeHair}(\mathit{tweety}) \; \}$$

Why does Tweety not fly? (2)

Contextual Abduction

[...] $\mathcal{O}$ is *contextually explained by* $\mathcal{E}$ *given* $\mathcal{P}$ iff $\mathcal{P} \cup \mathcal{E} \models_{wcs} L$ for all $L \in \mathcal{O}$,
and for all $A \leftarrow \top \in \mathcal{E}$ and $A \leftarrow \bot \in \mathcal{E}$
there exists an $L \in \mathcal{O}$, such that L restrictly depends on A.

The only contextual explanation for $\mathcal{O}$ is $\mathcal{E} = \{featherslikeHair(tweety) \leftarrow \top\}$.

lfp $(\Phi_{(\mathcal{P}'' \cup \mathcal{E})}) = \langle I^\top, I^\bot \rangle$ is

$$
\begin{array}{lcl}
I^\top & = & \{featherslikeHair(tweety), kiwi(tweety), ab_1(tweety)\}, \\
I^\bot & = & \{fly(tweety)\}.
\end{array}
$$

Tweety is an abnormal bird! Tweety is a kiwi!

Conclusions

- The new truth-functional operator, ctxt, fits quite well with the interpretation of negation as failure under three-valued semantics
- Some properties of the Φ operator don't hold anymore for contextual programs
  - The Φ operator is not monotonic
  - A least fixed point of the Φ operator is not always guaranteed
  - However, if $\mathcal{P}$ is acyclic, the existence of a least fixed point is guaranteed
- Contextual abduction allows us to specify relations between explanations and observations and prefer explanations to others depending on the context

How do the assumptions made for the development of contextual reasoning fit with the findings from Cognitive Science?