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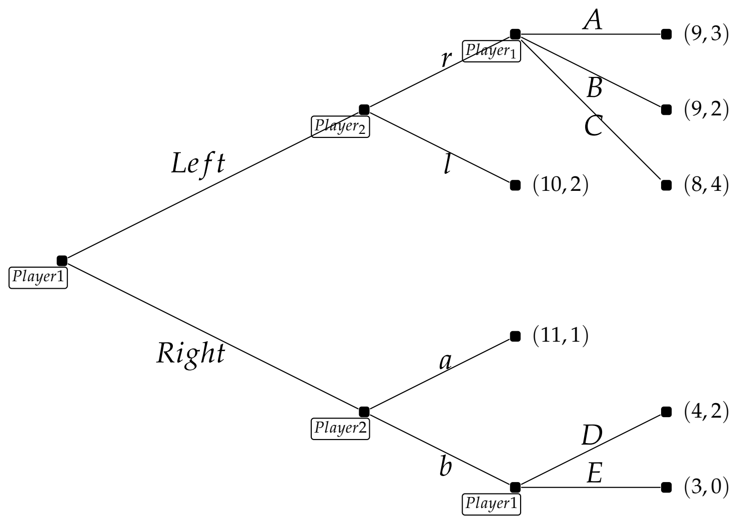
Algorithmic Game Theory

Summer Term 2024

exercises 3

29/04–03/05/2024

Problem 1.
Consider the following game tree.



(a) What is the number of strategies of player I and of player II?

(b) How many reduced strategies do they have? Recall that a reduced strategy in a game tree of a player specifies a move for every decision node of that player, expect for those moves that are unreachable due to an earlier move.

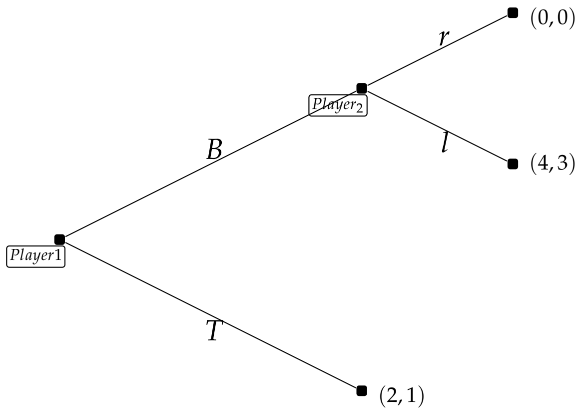
(c) Give the reduced strategic form of the game.

(d) What are the equilibria of the game in reduced strategies?

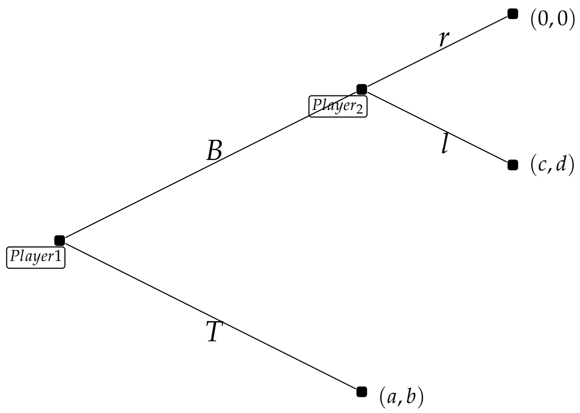
(e) What are the subgame-perfect equilibria of the game?

Problem 2.
Consider the following game trees.

Tree 1:



Tree 2:



(a) Find all equilibria for the game tree at the top by transforming the game into its strategic normal form. Which of these are subgame-perfect?

(b) In the game tree 2, the payoffs a, b, c, d are positive real numbers. For each of the following statements (i), (ii), (iii), decide if it is true or false, justifying your answer with an argument or counterexample.

(i) the game always has a subgame-perfect equilibrium (SPE);

(ii) the payoff to player II in any SPE is always at least as high as her payoff in any equilibrium;

(iii) the payoff to player I in any SPE is always at least as high as his payoff in any equilibrium.

Problem 3.

In 1981, Robert Rosenthal introduced the so-called **centipede game**. The game in extensive form can be described as follows

- At stage 1, player I chooses between move **R** and **D**.
  - If she chooses **D**, player 1 gets 1 and player 2 gets 0;
  - If she chooses **R**, the game goes to round 2.
- At stage 2, player 2 chooses between **r** and **d**.
  - If he chooses **d**, player 1 gets 0 and player 2 gets 2;
  - If he chooses **r**, the game moves to round 3.
- At stage 3, player 1 chooses between **R** and **D**.

  - If she chooses **D**, player 1 gets 3 and player 2 gets 1;
  - If she chooses **R**, the games moves to round 4.
- At stage 4, player chooses between **r** and **d**.
  - If he chooses **d**, player 1 gets 2 and player 2 gets 4.
  - If he chooses **r**, both players get 3.

Do the following

- Draw the tree representation of the game.
- Apply backward induction and find its outcome.
- Give the pure strategies of both players and the payoff matrix of the strategic normal form of the game.
- Find all (sub-game perfect) Nash equilibria.

After its introduction, the **centipede game** has been extensively studied in experiments. What do you think: Were the people taking part in the experiments more inclined to play the Nash equilibrium or to cooperate?

Problem 4.

Argue that in any sequential game, the backward induction strategy profile is in fact a Nash equilibrium.