

Exercise Sheet 8: Advanced L^AT_EX
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Please prepare exercise 8.4 for the exercise session on 2019-06-18.

Exercise 8.1. (continuation of exercise 7.2)

Typeset the following formulae:

$$\neg A \sqcap \exists R^- . A \sqcap (\leq 1 R) \sqcap \forall (R^-)^+ . (\exists R^- . A \sqcap (\leq 1 R)) \quad (1.1)$$

$$\pi_x(\leq n R) = \exists^{\leq n} y . R(x, y) = \exists y_1, \dots, y_n . \bigwedge_{i \neq j} y_i \neq y_j \supset \bigvee_i \neg R(x, y_i) \quad (1.2)$$

$$\text{Tree} \equiv \mu X . (\text{EmptyTree} \sqcup (\text{Node} \sqcap \leq 1 \text{child}^- \sqcap \exists \text{child} . \top \sqcap \forall \text{child} . X)) \quad (1.3)$$

$$(\mu X . C)_\rho^{\mathcal{I}} = \bigcap \{ \mathcal{E} \subseteq \Delta^{\mathcal{I}} \mid C_{\rho[X/\mathcal{E}]}^{\mathcal{I}} \subseteq \mathcal{E} \} \quad (1.4)$$

$$s \rightarrow_E t \text{ iff } \exists (l, r) \in E, p \in \text{Pos}(s), \sigma \in \text{Sub}. s|_p = \sigma(l) \text{ and } t = s[\sigma(r)]_p \quad (1.5)$$

$$\mathbb{K}[\mathfrak{C}]_r := (G \cup \mathfrak{C}_{\min}, M \cup \mathfrak{C}_{\max}, I_{\mathfrak{C}} \cap (G \cup \mathfrak{C}_{\min}) \times (M \cup \mathfrak{C}_{\max})) \quad (1.6)$$

$$0 = \int_{\{s_n(u) > \frac{1}{k} + \mathbf{E}^{A_n} u\}} (s_n(u) - \mathbf{E}^{A_n} u) d\mu \geq \frac{1}{k} \mu \left(\left\{ s_n(u) > \frac{1}{k} + \mathbf{E}^{A_n} u \right\} \right) \quad (1.7)$$

Exercise 8.2. Use bibtex to typeset a bibliography of all published literature referenced on exercise sheets 0–7. Make all references as complete as possible, and strive for consistency among the references.

Exercise 8.3. Typeset the following paragraph.

Lemma 8.2 (Yoneda) *Let \mathbf{C} be locally small. For any object $C \in \mathbf{C}$ and functor $F \in \text{Sets}^{\mathbf{C}^{\text{op}}}$ there is an isomorphism*

$$\text{Hom}(yC, F) \cong FC$$

which, moreover, is natural in both F and C .

Here:

(1) *the Hom is $\text{Hom}_{\text{Sets}^{\mathbf{C}^{\text{op}}}}$,*

(2) *naturality in F means that, given any $\vartheta : F \rightarrow G$, the following diagram commutes:*

$$\begin{array}{ccc} \text{Hom}(yC, F) & \xrightarrow{\cong} & FC \\ \text{Hom}(yC, \vartheta) \downarrow & & \downarrow \vartheta_C \\ \text{Hom}(yC, G) & \xrightarrow{\cong} & GC \end{array}$$

(3) *naturality in C means that, given any $h : C \rightarrow D$, the following diagram commutes:*

$$\begin{array}{ccc} \text{Hom}(yC, F) & \xrightarrow{\cong} & FC \\ \text{Hom}(yh, F) \uparrow & & \uparrow Fh \\ \text{Hom}(yD, F) & \xrightarrow{\cong} & FD \end{array}$$

Hint: You can use `tikz`¹ with the `tikz-cd` library to typeset the *commutative diagrams*, but several other packages are also available.

Exercise 8.4. (Homework)

Write a paper proving the *binomial theorem*:

Let $n \in \mathbb{N}$. Then

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}.$$

You may use any sources that you need, provided that you properly cite them. Make sure your paper includes an introduction, a conclusion, and all necessary preliminaries. Try to make your paper as easy to read as possible.

¹<https://github.com/pgf-tikz/pgf>