Cognitive Principles and Reasoning Clusters in Multinomial Process Tree: A Case Study for Human Syllogistic Reasoning

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April 11, 2018

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Multinomial Process Tree

Advantages:

- Intuitive way of representation
- High level description
- Takes guessing into account
- Not only observation of answers
- Quantitative mathematical metrics

Multinomial Process Tree

Multinomial Process Tree (MPT):

- Directed acyclic graph
- Finite set of response categories as leaves
- Finite set of cognitive processes as inner nodes
- Edges with parameter corresponding to probabilities

In our case, the finite set of response categories are the 9 possible conclusions (Aac, Eac, Iac, ...)
The finite set of cognitive processes are groups of one or more cognitive principles

Multinomial Process Tree

A Multinomial Process Tree is composed of two parts:

- **Reasoning part**: sub-tree whose nodes are results of a reasoning process of an individual
- **Guessing part**: sub-tree whose nodes are not cognitive processes. The set of leaves correspond to the set of conclusions, which are made by guess

Criteria for an Evaluation

- Goodness of Fit
- Akaike Information Criterion
- Bayesian Information Criterion
- Root Mean Square Error
- Fisher Information Approximation

Representation of the Mental Model Theory

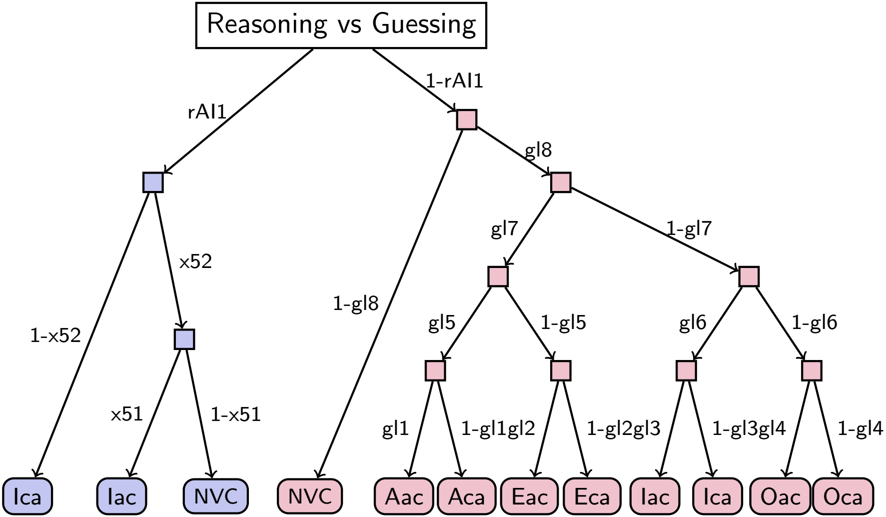


Figure: MPT for the syllogism AI1

Representation of the Mental Model Theory

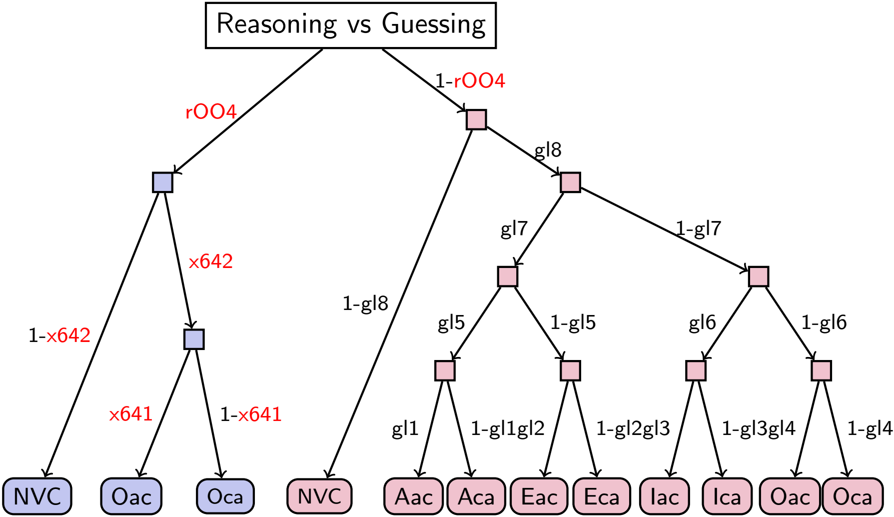


Figure: MPT for the syllogism OO4

Cognitive Principles

Basic principles are assumed to be used by all reasoners

- Quantified statements as conditionals
- Licenses for inferences
- Existential import
- Unknown generalization

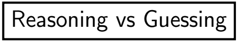
Advanced principles are not necessarily applied by all reasoners

- Search for alternative conclusions (abduction)
- Contraposition
- Deliberate Generalization

Representation with the same Multinomial Process Tree

Figure: General MPT

Representation with the same Multinomial Process Tree



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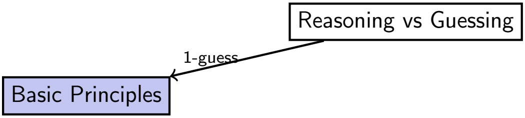

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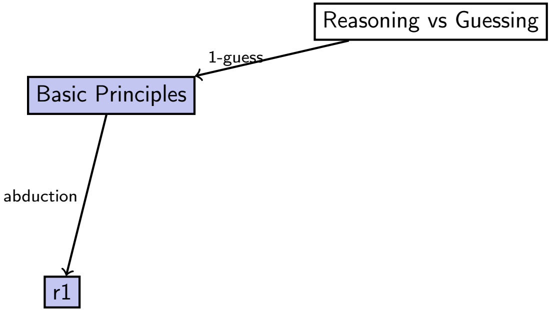

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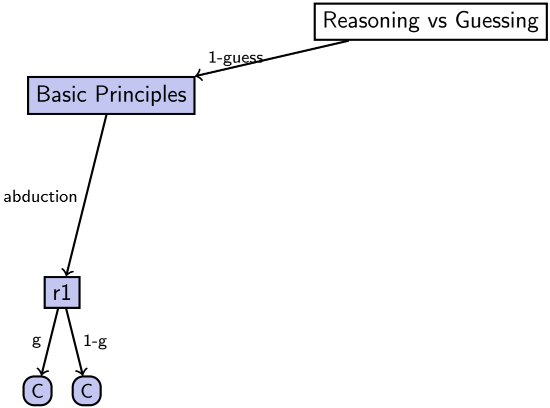

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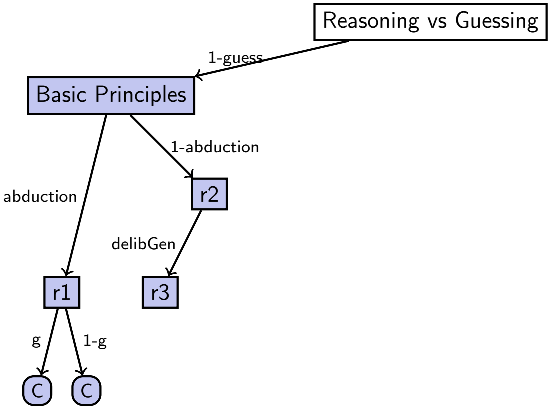

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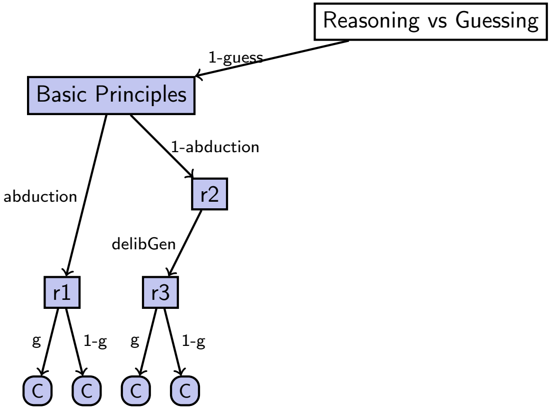

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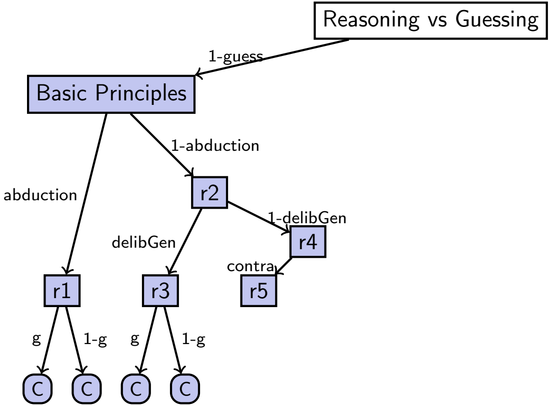

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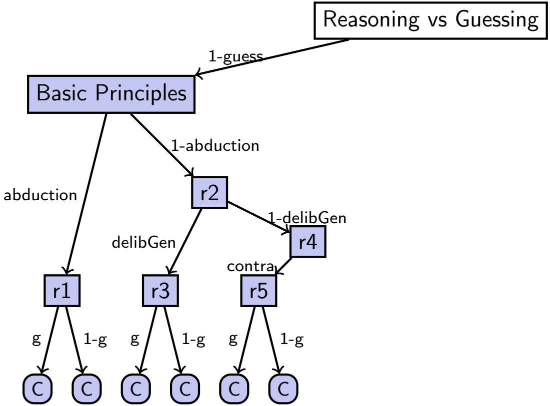

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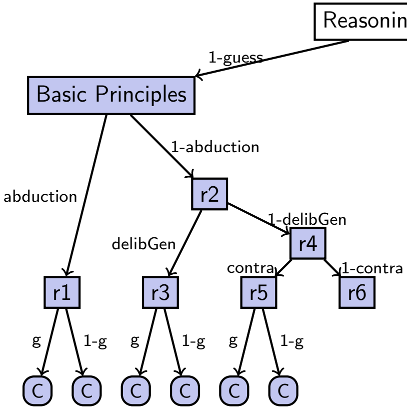

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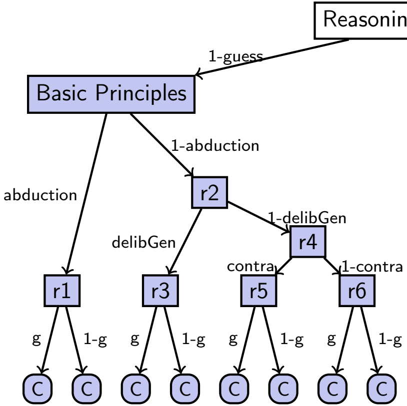

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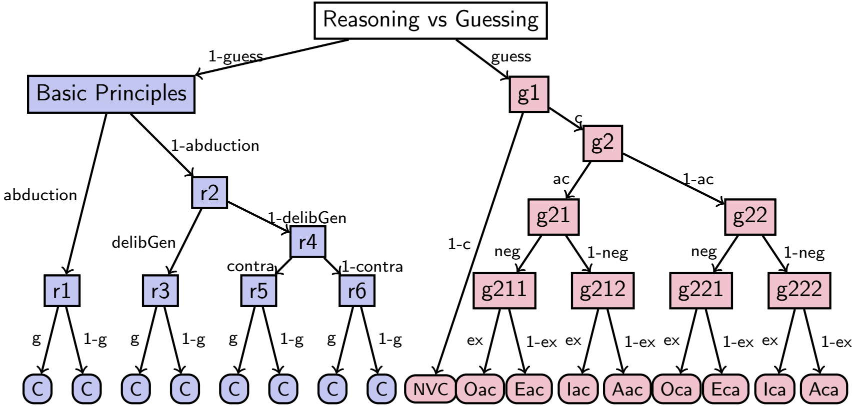

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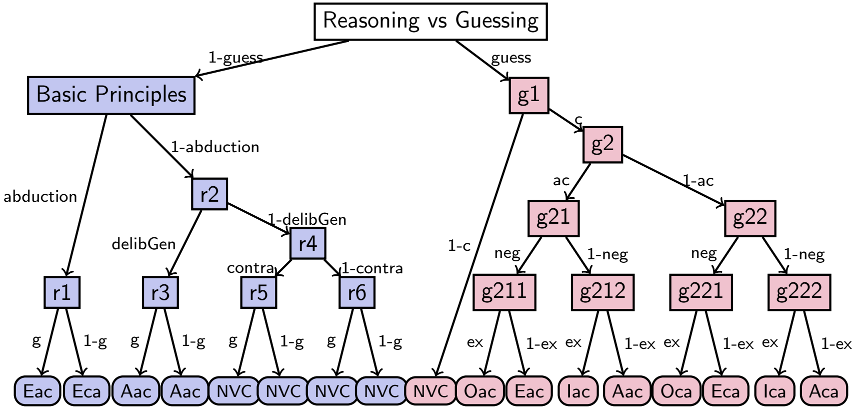

Figure: MPT for the syllogism AI1 with setup 1

Filtering approach

Filtering approach: filters out conclusions of the original guessing tree that are unlikely according to some heuristic strategies

Setup 2: Matching Strategy

Order defined on the moods, from the most to the least conservative quantifier:

$$\mathbf{E} > \mathbf{O} = \mathbf{I} > \mathbf{A}$$

In the guessing part of each MPT:

- branches that lead to a conclusion with a mood less conservative than the higher conservative mood of the premise get very low probabilities

Setup 2: Matching Strategy

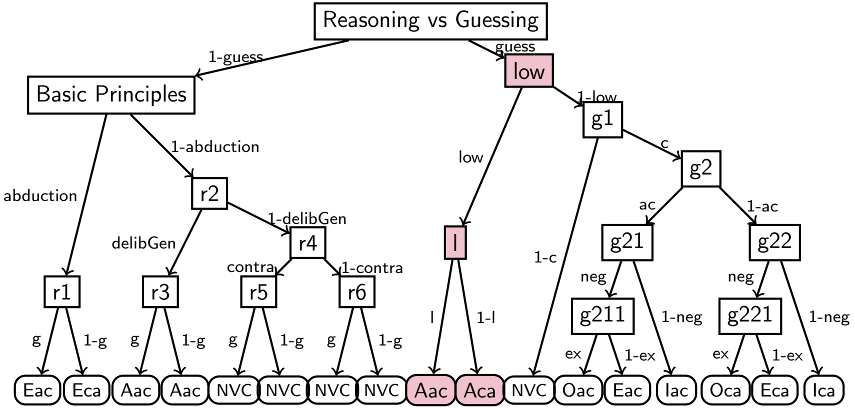

Figure: MPT for the syllogism AI1 with setup 2

Setup 3: Biased Conclusions in Figure 1

For syllogistic premises of figure 1:

- Only branches leading to the conclusion Xac is given a high probability
- $X \in \{A, I, O, E\}$: the most conservative mood from the pair of premises under the matching strategy
- All other branches get a low probability
- No change for other figures

Setup 3: Biased Conclusions in Figure 1

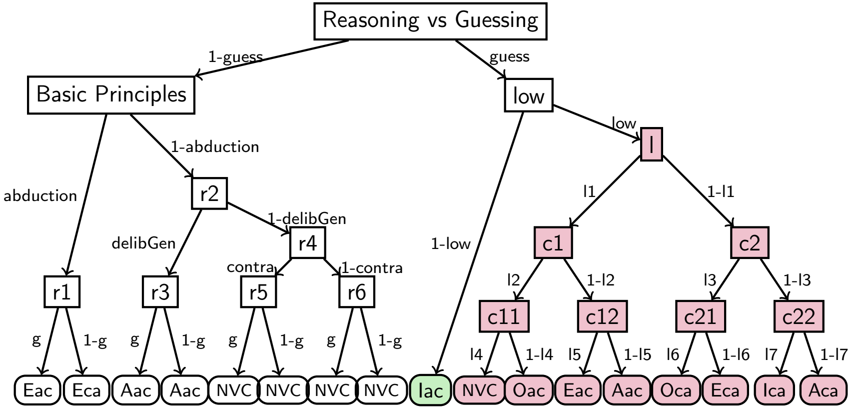

Figure: MPT for the syllogism AI1 with setup 3

Setup 4: Combination of Filters

- For syllogistic premises of figure 1 (order a-b b-c): *biased conclusions in figure 1* is applied
- For other syllogistic premises: *matching strategy* is applied

Related Work

- Heuristic Theories of Syllogistic Reasoning
  - Atmosphere theory
  - Matching theory
  - Illicit Conversion
  - Probability heuristics (PHM)
- Theories Based on Formal Rules
  - PSYCOP model
- Model Based Theories
  - Verbal models theory
  - Mental model theory

Overview of Predictions

Syllogistic Premises	AI1	AE1	IE3
Valid Conclusions	NVC ~~Iac Ica~~	Eac Eca Oac Oca	Oac ~~Eca NVC~~
Participants	Iac (35%) Ica (35%) NVC (15%)	Eac (50%) Eca (40%)	Oac (35%) Eca (30%) NVC (20%)
Clustering WCS	Iac (70%) NVC (19%) ~~Ica~~	Iac (31%) Eac (69%) ~~Eca~~	Oac (28%) Ica (15%) NVC (18%) ~~Eca~~
Atmosphere	Iac (35%) Ica (35%) NVC (16%)	Eac (50%) Eca (40%)	Oac (35%) NVC (29%) ~~Eca~~
Conversion	Iac (35%) Ica (35%) ~~NVC~~	Eac (50%) Eca (40%)	Oac (35%) Oca (14%) ~~Eca NVC~~
PSYCOP	Oac (16%) Oca (17%) NVC (19%) ~~Iac Ica~~	Eac (50%) Eca (40%)	Oac (35%) NVC (14%) ~~Eca~~
Matching	Iac (35%) Ica (35%) NVC (16%)	Eac (50%) Eca (40%)	Eca (30%) NVC (29%) ~~Oac~~
Mental Model Theory	Iac (35%) Ica (34%) NVC (15%)	Eac (50%) Eca (40%)	Oac (35%) Eca (30%) NVC (20%)
Mental Model Theory	Iac (35%) ~~Ica NVC~~	Eac (50%) ~~Eca~~	Oac (35%) Eca (30%) ~~NVC~~
PHM	Iac (35%) NVC (27%) ~~Ica~~	Eac (50%) ~~Eca~~ NVC (20%)	Eca (30%) NVC (29%) ~~Oac~~
Verbal	Iac (35%) ~~Ica NVC~~	Eac (50%) ~~Eca~~	Oac (35%) NVC (20%) ~~Eca~~

Comparison with Data of Ragni et al. (2016)

	Model	k	G^2	AIC	BIC	RMSE	FIA
Mental Model	MMT	235	50.45	506	1014	0.12	235
Same parameters	Setup 1	13	62.45	88.45	116.52	0.13	13
	Setup 2	15	63.58	93.58	125.96	0.15	15
	Setup 3	14	59.21	87.21	117.43	0.13	14
	Setup 4	15	67.75	98.75	130.13	0.15	15

Comparison with Data of Khemlani & Johnson-Laird (2012)

	Model	k	G^2	AIC	BIC	RMSE	FIA
Mental Model	MMT	235	4.85	474.85	968.81	0.02	235
Same parameters	Setup 1	13	73.41	99.41	126.73	0.14	13
	Setup 2	15	56.99	86.99	118.52	0.12	15
	Setup 3	14	61.27	89.27	118.68	0.12	14
	Setup 4	15	53.99	83.99	115.52	0.11	15

Comparison with other Approaches

Model	k	G^2	AIC	BIC	RMSE	FIA
MMT1	235	50.45	506	1014	0.12	235
Clustering WCS	13	62.45	88.45	116.52	0.13	13
Atmosphere	136	27.00	299.00	592.61	0.16	136
Matching	200	23.38	423.38	855.16	0.17	NA
Conversion	92	42.06	226.06	424.68	0.15	92
PHM	168	33.63	369.63	732.32	0.16	NA
PSYCOP	131	38.91	300.91	583.72	0.15	NA
Verbal Model	128	35.07	291.07	567.41	0.15	128
MMT2	235	14.95	484.95	992.29	0.17	232

Conclusion

Open questions:

- Reconsideration of the 4 clusters
- Emphasis placed:
  - not in fitting evaluation criteria
  - in a new way of representation

Thank you for your attention

Preliminaries - Syllogistic Reasoning

Mood	First-order logic	Short
affirmative universal	$\forall X(a(X) \rightarrow b(X))$	Aab
affirmative existential	$\exists X(a(X) \wedge b(X))$	Iab
negative universal	$\forall X(a(X) \rightarrow \neg b(X))$	Eab
negative existential	$\exists X(a(X) \wedge \neg b(X))$	Oab

Figure: The moods and their formalization.

	1st Premise	2nd Premise
Fig. 1	a-b	b-c
Fig. 2	b-a	c-b
Fig. 3	a-b	c-b
Fig. 4	b-a	b-c

Figure: The four figures.

Preliminaries - Syllogistic Reasoning Task

Some artists are bakers.
No chemists are bakers. (IE3)

Preliminaries - Syllogistic Reasoning Task

Some artists are bakers.
No chemists are bakers. (IE3)

64 distinct pairs of premises for the syllogistic reasoning task:

- 4 possible moods for the first premise
- 4 possible moods for the second premise
- 4 figures

Preliminaries - Syllogistic Reasoning Task

Some artists are bakers.
No chemists are bakers. (IE3)

64 distinct pairs of premises for the syllogistic reasoning task:

- 4 possible moods for the first premise
- 4 possible moods for the second premise
- 4 figures

9 possible answers with respect to artists and chemists:

All artists are chemists.	All chemists are artists.
Some artists are chemists.	Some chemists are artists.
No artists are chemists.	No chemists are artists.
Some artists are not chemists.	Some chemists are not artists.
No Valid Conclusion	

Possible Variation of Parameters

Two versions for each strategy:

- Same parameters in the reasoning part of each tree
- Different parameters in the reasoning part of each tree

Related Work

- Heuristic Theories of Syllogistic Reasoning
  - Atmosphere theory
  - **Matching theory**
  - Illicit Conversion
  - Probability heuristics
- Theories Based on Formal Rules
  - **PSYCOP model**
  - Verbal substitution
  - Source founding theory
  - Monotonicity theory
- Model Based Theories
  - Euler circles
  - **Venn diagrams**
  - Verbal models theory
  - Mental model theory

Comparison with Data of Ragni et al. (2016)

	Model	k	G^2	AIC	BIC	RMSE	FIA
Mental Model	MMT	235	50.45	506	1014	0.12	235
Same parameters	Setup 1	13	62.45	88.45	116.52	0.13	13
	Setup 2	15	63.58	93.58	125.96	0.15	15
	Setup 3	14	59.21	87.21	117.43	0.13	14
	Setup 4	15	67.75	98.75	130.13	0.15	15
Different parameters	Setup 1	265	48.78	578.78	1150.88	0.12	NA
	Setup 2	267	49.25	588.25	1159.68	0.16	NA
	Setup 3	266	46.78	578.78	1153.04	0.13	NA
	Setup 4	267	53.29	587.29	1163.71	0.16	NA

Comparison with Data of Khemlani & Johnson-Laird (2012)

	Model	k	G^2	AIC	BIC	RMSE	FIA
Mental Model	MMT	235	4.85	474.85	968.81	0.02	235
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	Setup 3	14	61.27	89.27	118.68	0.12	14
	Setup 4	15	53.99	83.99	115.52	0.11	15
Different parameters	Setup 1	265	53.34	583.34	1140.36	0.11	NA
	Setup 2	267	37.71	571.71	1132.94	0.09	NA
	Setup 3	266	43.36	575.36	1134.49	0.09	NA
	Setup 4	267	34.89	568.89	1130.12	0.08	NA

Competition - Algorithmic part

Participants	Method	RMSE
Antonis Kakas (1)	Argumentation	0.067
Antonis Kakas (2)	Argumentation	0.074
Sangeet Khemlani	mReasoner (Mental Models)	0.145
Frieder Stolzenburg	Set Theory	0.161
Our contribution	Weak Completion Semantics	0.166

Table: Results of the Competition

Goodness of Fit (G^2)

Analyze the distance between predicted and observed responses frequencies

$$G^2 = 2 \sum_{t=1}^{T} \sum_{j=1}^{J_t} n_{j,t} [ln(n_{j,t}) - ln(N_t p_{j,t})] \quad (1)$$

- $n_{j,t}$: frequency of response category j in tree t
- $N_t = \sum_{j=1}^{J_t} n_{j,t}$
- $p_{j,t}$: probability of response category j in tree t

Likelihood Function (L)

Describes the plausibility of a parameter value given certain data

$$L = p(x|\Theta, M) \tag{2}$$

- x: observed data
- Θ: parameters
- M: model

Akaike Information Criterion (AIC)

Compares the quality of each model relative to the given data and takes into account the number of parameters

$$AIC = 2k - 2ln(L) \tag{3}$$

- k: number of parameters
- L: maximum value of the likelihood function

Bayesian Information Criterion (BIC)

Compares the model to the given data and punishes more a high number of parameters

$$BIC = ln(n)k - 2ln(L) \tag{4}$$

- n: number of observations
- k: number of parameters
- L: maximum value of the likelihood function

Root Mean Square Error (RMSE)

Measures the differences between values predicted by the model and values observed: square root of the average of squared errors, the effect of each error is proportional to the size of the squared error

$$RMSE = \sqrt{\frac{\sum_{i=1}^{k}(y_i' - y_i)^2}{k}} \quad (5)$$

- k: number of parameters
- y_i': predicted values
- y_i: observed values

Fisher Information Approximation (FIA)

Provides a more precise quantification of model flexibility: observes flexibility differences in models that have the same number of parameters

$$FIA = \frac{1}{2}G^2 + \frac{k}{2}ln(\frac{N}{2\pi}) + ln(\int \sqrt{detI(\Theta)}d\Theta) \qquad (6)$$

- G^2: goodness of fit
- k: number of parameters
- $N = \sum_{t=1}^{T} N_t$
- $N_t = \sum_{j=1}^{J_t} n_{j,t}$
- $n_{j,t}$: frequency of response category j in tree t
- $I(\Theta)$: Fisher information matrix