

Hannes Strass

(based on slides by Bernardo Cuenca Grau, Ian Horrocks, Przemysław Wałęga)

Faculty of Computer Science, Institute of Artificial Intelligence, Computational Logic Group

Introduction and Overview

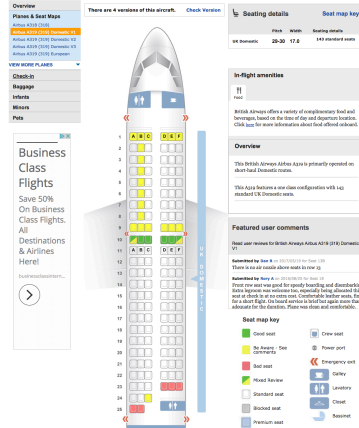
Lecture 1, 14th Oct 2024 // Foundations of Knowledge Representation, WS 2024/25

Applications often need to represent knowledge

- seats on an aeroplane

British Airways Seat Maps

Airbus A319 (319) Domestic V1



Applications often need to represent knowledge

- seats on an aeroplane
- account transactions

YourDosh Bank
16 High Street, Anytown, Anyshire Y299 1XY

Mr John Smith
5 Any Road
Randomford
Anyshire
YZ98 5XY

Account name: Mr John Smith
Sort code: 63-61-33
Account number: 99988877

Your current account statement:
1 February to 1 March 2011
Page 1 of 1

Your account summary

Balance at 1 February: £312.34
Total money in: £300.00
Total money out: £343.02
Balance at 1 March: £30.68 OD

Date	Description	Money out	Money in	Balance
	Balance brought forward			312.34
1 February	Card payment - High St Petrol Station	24.50		287.84
	Direct debit - Green Mobile Phone Bill	20.00		267.84
3 February	Cash withdrawal - YourDosh, Anytown High Street, timed 17.35-21 Jan	30.00		237.84
8 February	Cheque 50568	22.95		214.89
11 February	BACS - KleanKars, ref J55-999		300.00	514.89
16 February	Standing order - Rent to J Jones Cash withdrawal - WadBank, Randomford, timed 09.52 14 Feb	300.00		214.89
17 February	Card Payment - High St Petrol Station	40.00		67.14
	Direct debit - Home Insurance	23.00		44.14
19 February	Online transfer to A/C 1116932, Sort 01-92-10 ref: Mum	34.14		10.00
21 February	Card payment - QuidsyGifts	15.00		5.00 OD
24 February	Balance carried forward			5.68 OD
28 February	Card payment - QuidsyGifts	25.00		30.68 OD
1 March	Interest 29 Jan A/C 99988877		00.68	30.68 OD
	Charges 29 Jan A/C 99988877	25.00		30.68 OD
	Balance carried forward			30.68 OD

Applications often need to represent knowledge

- seats on an aeroplane
- account transactions
- tall buildings

British Airways Seat Maps

A320neo (119) Domestic V1










Name	City	State	Height	Feet	Floors	Completed
Federation Tower: East Tower	Moscow	Russia	373.7	1,226	95	2016
OKO: South Tower	Moscow	Russia	354.1	1,162	85	2015
Mercury City Tower	Moscow	Russia	338.8	1,112	75	2013
The Shard ^[1]	London	United Kingdom	309.7	1,017	87	2012
Eurasia ^[2]	Moscow	Russia	308.9	1,014	72	2014
CoC: Moscow Tower ^[3]	Moscow	Russia	301.6	990	76	2010
Skyland Istanbul 1 ^{[4][5]}	Istanbul	Turkey	293.1	932	65	2017
Skyland Istanbul 2 ^{[4][5]}	Istanbul	Turkey	293.1	932	65	2017
Metropol Istanbul ^[6]	Istanbul	Turkey	280	919	66	2017
Emaar Square	Istanbul	Turkey	280	920	62	2018
Naberezhnaya Tower C	Moscow	Russia	268.4	881	59	2007
Triumph Palace ^[7]	Moscow	Russia	264.1	867	57	2005
Commerzbank Tower ^{[8][9]}	Frankfurt	Germany	258.7	848	58	1997
CoC: Saint Petersburg Tower ^[10]	Moscow	Russia	256.9	843	65	2010
Messezentrum	Frankfurt	Germany	256.5	842	55	1990
Nurel Life ^{[11][12]}	Istanbul	Turkey	252	827	60	2017
Torre de Cristal	Madrid	Spain	249	817	45	2008
Torre Cepsa	Madrid	Spain	248.3	815	45	2008
Evolution Tower ^[13]	Moscow	Russia	245.9	807	53	2014
OKO: North Tower ^[14]	Moscow	Russia	245	804	49	2014
Federation: West Tower	Moscow	Russia	243.2	798	62	2007
Main building of Moscow State University	Moscow	Russia	240	787	38	1953
Imperia Tower	Moscow	Russia	238.7	783	60	2011
Palace of Culture and Science	Warsaw	Poland	237	777	43	1955
Torre PwC	Madrid	Spain	236	774	52	2008
1 Canada Square	London	United Kingdom	235	771	50	1991
Istanbul Sapphire ^{[15][16]}	Istanbul	Turkey	234.9	770	54	2010
Tour First	Paris ^B	France	231	758	56	2011
Unicredit Tower	Milan	Italy	231	758	35	2011
Heron Tower	London	United Kingdom	230	755	46	2011

Applications often need to represent knowledge

- seats on an aeroplane
- account transactions
- tall buildings

and to answer questions

- seats available on flight?

 SAS	19:05 LHR	— — —	12:00 OSL (+1)	15h 55m	£187 Opodo	View Deal
 SAS	17:20 OSL	— — —	18:50 LHR (+1)	26h 30m	5 more £187	Share Watch
See details						
Add a hotel with Expedia Omega £190 tripota £198						
 SWISS	12:05 LHR	— — —	19:00 OSL	5h 55m	£230 Gatagate	View Deal
 Lufthansa	06:00 OSL	— — —	14:40 LHR	9h 40m	3 more £243	Share Watch
See details						
Add a hotel with Expedia mytrip.com £237 Omega £237						
Fly for less to Oslo with Austrian Airlines						
 Enjoy a relaxing flight with delightful Austrian service and hospitality. Book now at austrian.com <small>Austrian.com Sponsored</small>						
 Lufthansa	06:30 LHR	— — —	12:00 OSL	4h 30m	£241 Gatagate	View Deal
 Lufthansa	06:00 OSL	— — —	14:40 LHR	9h 40m	4 more £247	Share Watch
See details						
Add a hotel with Expedia BudgetAir £244 mytrip.com £247						
 Lufthansa	17:30 LHR	— — —	23:10 OSL	4h 40m	£241 Gatagate	View Deal
 Lufthansa	06:00 OSL	— — —	14:40 LHR	9h 40m	3 more £254	Share Watch
See details						
Add a hotel with Expedia mytrip.com £247 Omega £247						

Applications often need to represent knowledge

- seats on an aeroplane
- account transactions
- tall buildings

and to answer questions

- seats available on flight?
- can afford to pay rent?

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Applications often need
to represent knowledge

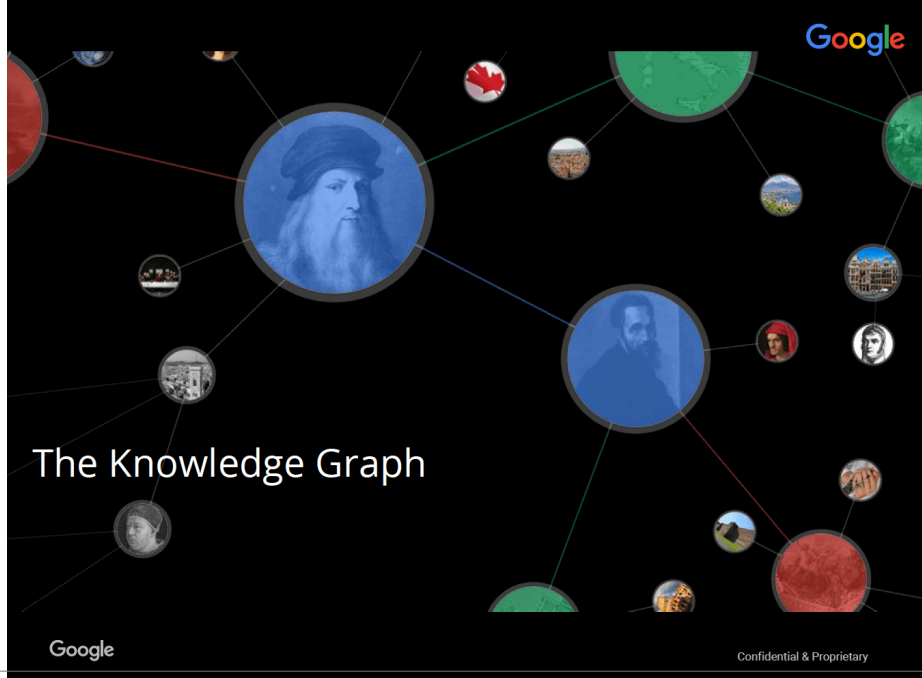
- seats on an aeroplane
- account transactions
- tall buildings

and to answer
questions

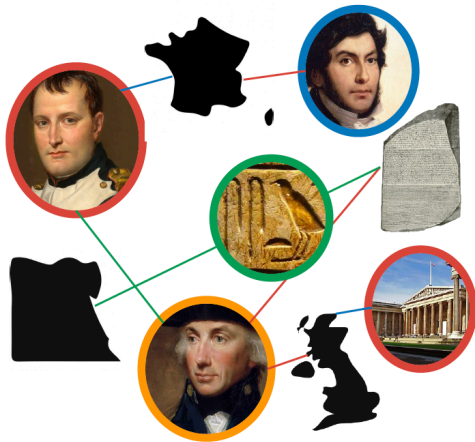
- seats available on
flight?
- can afford to pay rent?
- tallest building in
Europe?



What kind of representation?



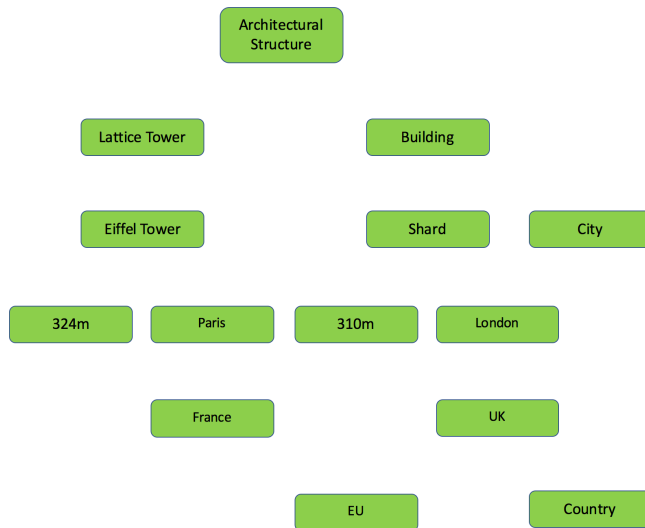
The Knowledge Graph

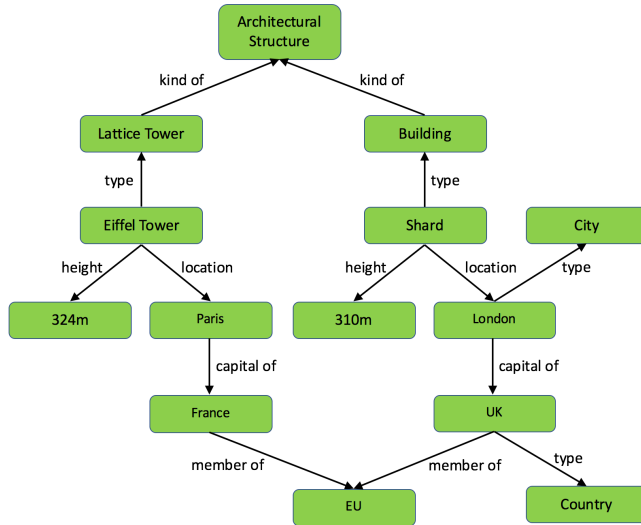


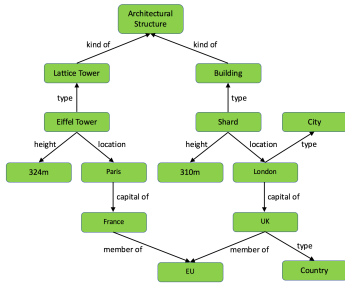
Google

The Knowledge Graph is a comprehensive collection of real-world entities (people, places, things, and concepts) along with relationships and factual attributes that describe them.

Confidential & Proprietary

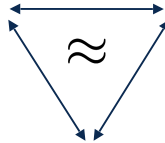






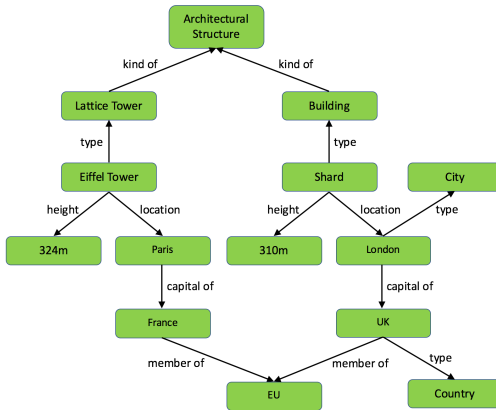
Architectural Structure			
name	location	height	kind
Shard	London	310m	Building
Eiffel Tower	Paris	324m	Lattice Tower
...

City		member of	
name	capital of	country	organisation
London	UK	France	EU
Paris	France	UK	EU
...



Building(Shard)
 City(London)
 location(Shard,London)
 height(Shard,310m)
 capitalOf(London,UK)

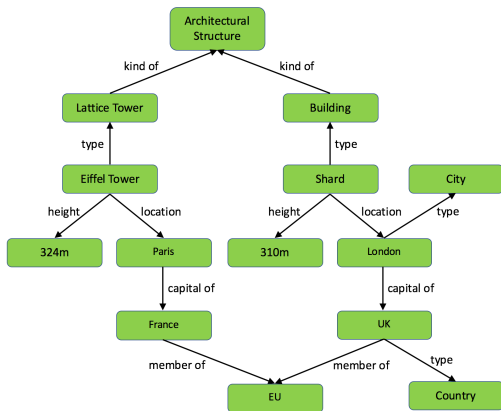
...



Reasoning is the process of **answering queries** w.r.t. the represented knowledge

What is the **height** of the **Eiffel Tower**?

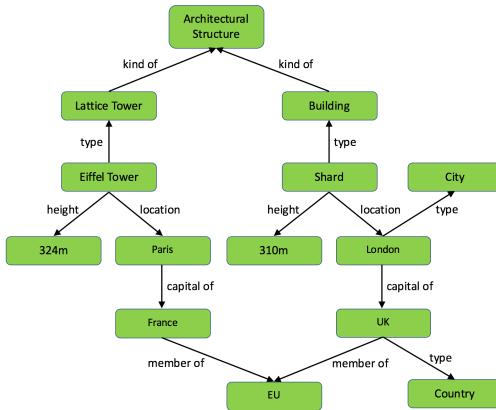
```
SELECT ?x
WHERE { EiffelTower height ?
x. }
```



Reasoning is the process of **answering queries** w.r.t. the represented knowledge

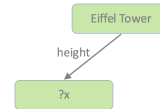
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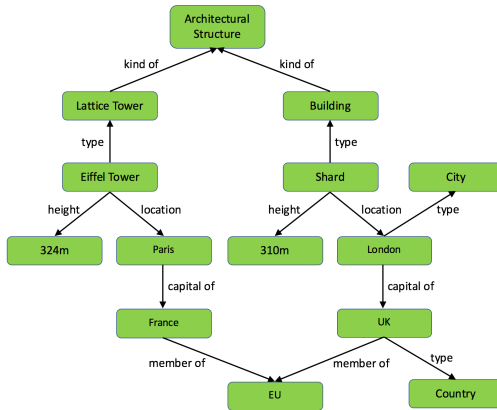




- Reasoning is the process of **answering queries** w.r.t. the represented knowledge

- What is the **height** of the **Eiffel Tower**?





- Reasoning is the process of **answering queries** w.r.t. the represented knowledge
- What is the **height** of the **Eiffel Tower**?
- 324m**

About 41,100,000 results (0.68 seconds)

Eiffel Tower / Height

300 m, 324 m to tip



People also search for



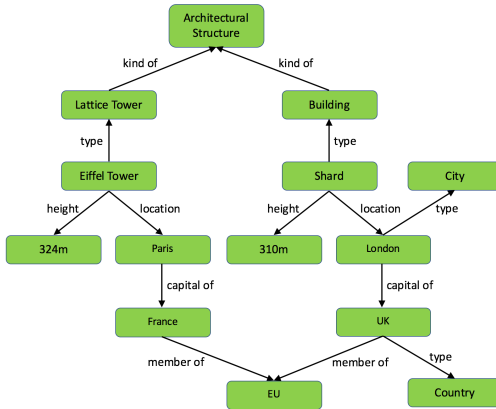
Burj Khalifa
828 m



Statue of
Liberty
93 m

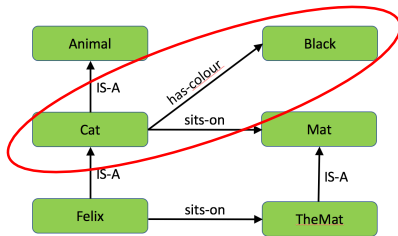


Leaning Tower
of Pisa
58 m



- What is the difference between a **Building**, a **Lattice Tower** and an **Architectural Structure**?
- Is the **Eiffel Tower** a **Building**; is it an **Architectural Structure**?
- Special meaning of, e.g., **type** and **kind of** edges?

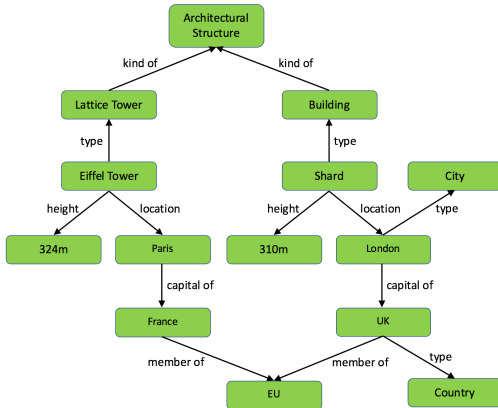
- **Semantics:** the study of meaning



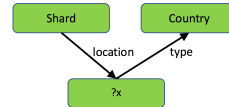
[Quillian, 1967]

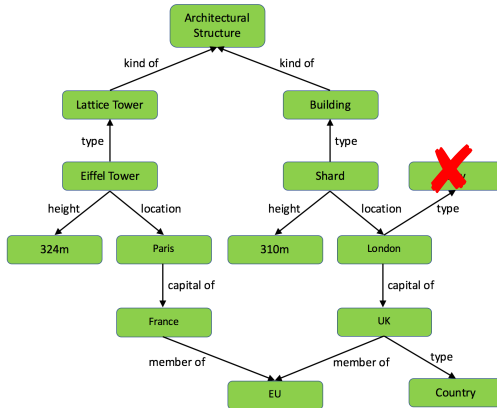


(Precise) semantics needed in order to define what (correct) query answers should be



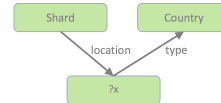
- What **country** is the **Shard** located in?

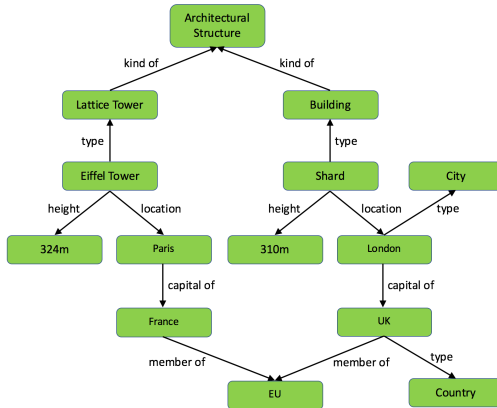




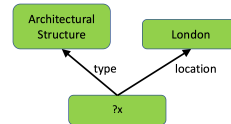
- What **country** is the **Shard** located in?

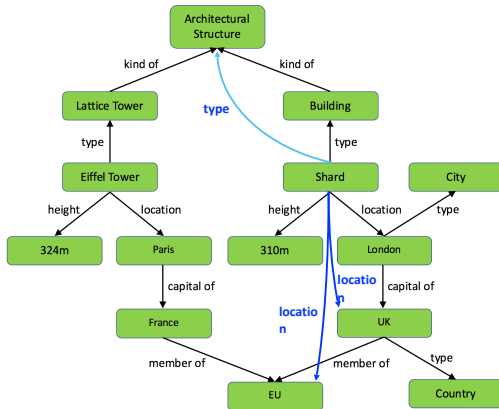
• ???

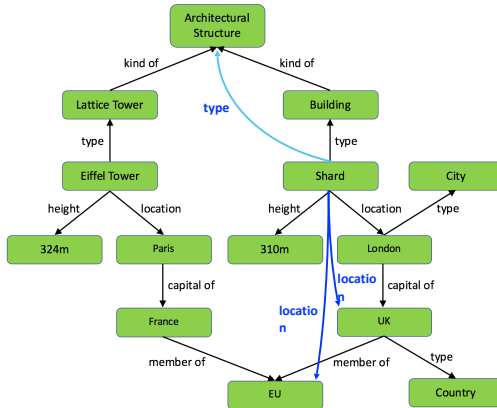




- What **Architectural Structures** are **located** in **London**?







- Every Building located in London is also located in
 - UK
 - EU
 - England
 - Northern Hemisphere
 - ...
- Need to add a **very** large number of edges

A city that is the capital of a country is a (geographical) part of that country[†]

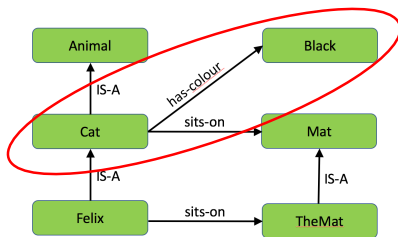
A thing that is located in a city that is a (geographical) part of a country is also located in that country

A thing that is located in a country that is a member of a supranational union is also located in that supranational union

[†] Part-whole relationships are complicated! They are the subject of a whole field of study in logic and philosophy: mereology

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- **Semantics:** the study of meaning



[Quillian, 1967]



(Precise) semantics needed in order to define what (correct) query answers should be



Supreme genus:

Differentiae:

Subordinate genera:

Differentiae:

Subordinate genera:

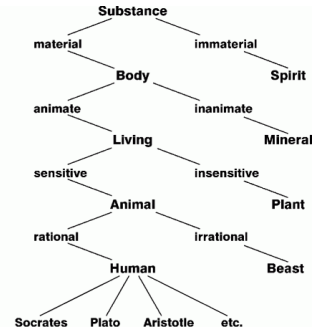
Differentiae:

Proximate genera:

Differentiae:

Species:

Individuals:



“All men are mortal, all Greeks are men,
therefore all Greeks are mortal”
(syllogism)

Modern KR languages are often based on logic

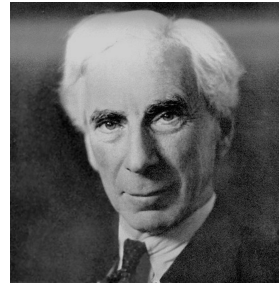
Typically (subsets of) First Order Predicate Calculus



Gottlob Frege

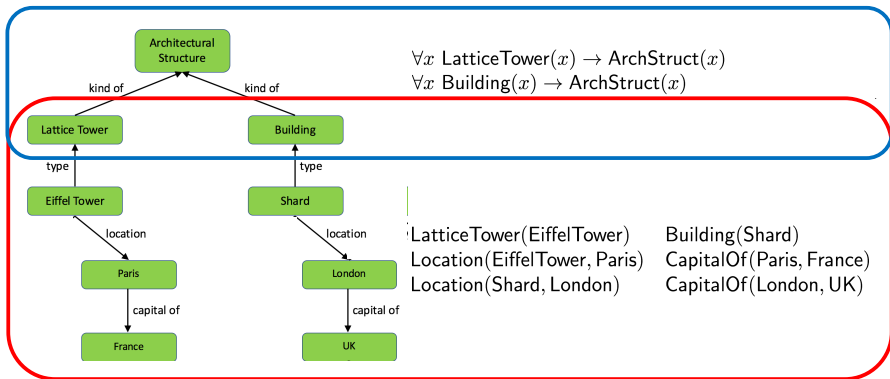


Charles Sanders
Peirce



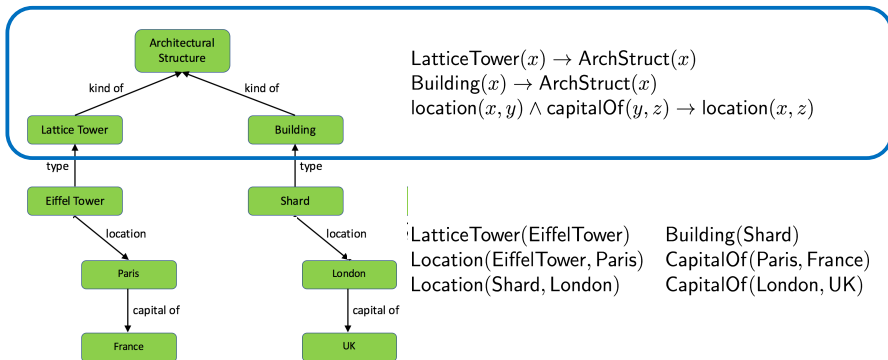
Bertrand Russell

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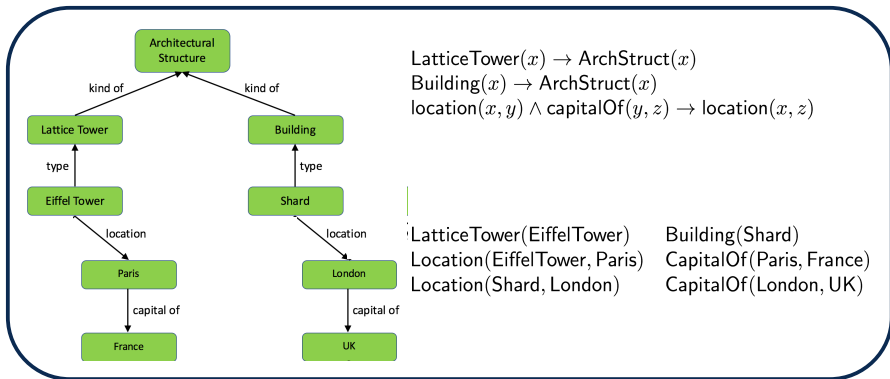
facts / data / (RDF) graph

ontology / conceptual schema



facts / data / (RDF) graph

ontology / conceptual schema



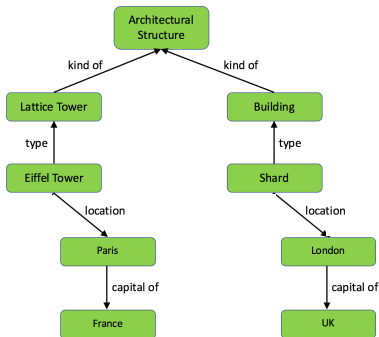
facts / data / (RDF) graph

ontology / conceptual schema

knowledge

base

32



$\text{LatticeTower}(x) \rightarrow \text{ArchStruct}(x)$
 $\text{Building}(x) \rightarrow \text{ArchStruct}(x)$
 $\text{location}(x, y) \wedge \text{capitalOf}(y, z) \rightarrow \text{location}(x, z)$

$\text{LatticeTower}(\text{EiffelTower})$	$\text{Building}(\text{Shard})$
$\text{Location}(\text{EiffelTower}, \text{Paris})$	$\text{CapitalOf}(\text{Paris}, \text{France})$
$\text{Location}(\text{Shard}, \text{London})$	$\text{CapitalOf}(\text{London}, \text{UK})$

$\mathcal{K} \models \text{ArchStruct}(\text{EiffelTower})$?

True
 True
 True
 True
 False

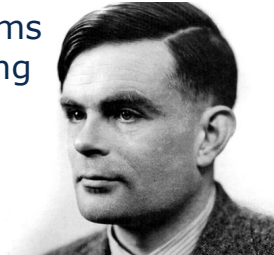
Devise **algorithms** that compute query answers
 E.g., using **natural deduction** rules:

$$\begin{array}{c}
 \frac{\forall x P(x) \rightarrow R(x)}{\forall x Q(x) \rightarrow R(x)} \quad \frac{\forall x P(x) \rightarrow Q(x)}{\forall x P(x) \rightarrow Q(x)} \quad \frac{\forall x \text{Greek}(x) \rightarrow \text{Mortal}(x)}{\forall x \text{Man}(x) \rightarrow \text{Mortal}(x)} \quad \frac{\forall x \text{Greek}(x) \rightarrow \text{Man}(x)}{\forall x \text{Greek}(x) \rightarrow \text{Man}(x)} \\
 \\
 \frac{Q(a)}{\forall x P(x) \rightarrow Q(x)} \quad P(a) \quad \frac{\text{ArchStruct}(\text{EiffelTower})}{\forall x \text{Building}(x) \rightarrow \text{ArchStruct}(x)} \quad \text{Building}(\text{EiffelTower})
 \end{array}$$

Can check/prove algorithms are **sound** and **complete** w.r.t. semantics

Turing showed that some problems cannot be completely solved using standard computational model

- halting problem
- **FOL entailment problem**



Even if decidable, reasoning might be of inherently **high complexity** and so take an **infeasibly long time**

“Scruffy” approach:

Ad-hoc representation

Efficient but (at least) incomplete algorithms

- ✓ Can use arbitrarily powerful representation
- ✓ Favourable scalability properties
- ✗ Incomplete answers
 - ✗ Degree of incompleteness unknown
 - ✗ Incompleteness can easily become unsoundness

“Neat” approach:

Study KR languages to find appropriate balance of expressive power and computability

Design algorithms that work well in **typical** cases

Develop highly optimised implementations

- ✓ Precisely defined semantics
- ✓ Formal properties well understood
- ✓ Sound and complete reasoning
- ✗ Limited representation power
- ✗ Optimisations may not offer robust scalability

Family of logic-based KR languages

Most are decidable subsets of FO logic

Provide a range of different constructors

- Booleans (and, or, not)
- Restricted forms of quantification (exists, forall)
- Counting (atmost, atleast)
- ...

Decidability/complexity and (efficient) algorithms
known for many combinations of constructors

Highly optimised implementations for various
“sweet spot” languages



Complexity of reasoning in Description Logics

Note: the information here is (always) incomplete and **updated** often

Base description logic: **Attributive Language with Complements**

$\mathcal{ALC} ::= \perp \mid A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \exists R.C \mid \forall R.C$



Concept constructors:

- ☐ F – functionality¹: $(\leq 1 R)$
- ☒ N – (unqualified) number restrictions: $(\geq n R), (\leq n R)$
- ☐ Q – qualified number restrictions: $(\geq n R.C), (\leq n R.C)$
- ☒ O – nominals: $\{a\}$ or $\{a_1, \dots, a_n\}$ ("one-of" constructor)
- ☐ μ – least fixpoint operator: $\mu X.C$
- ☐ $R \sqsubseteq S$ – role-value-maps
- ☐ $f = g$ – agreement of functional role chains ("same-as")

TBox is *internalized* in extensions of $\mathcal{ALC/O}$, see [76, Lemma 4.12], [54, p.3]

- ☐ Empty TBox
- ☐ Acyclic TBox ($A \sqsubseteq C$, A is a concept name; no cycles)
- ☐ General TBox ($C \sqsubseteq D$ for arbitrary concepts C and D)

Role constructors:

- ☒ I – role inverses: R^-
- ☐ \cap – role intersection³: $R \sqcap S$
- ☐ \cup – role union: $R \sqcup S$
- ☐ \neg – role complement: \bar{R}
- ☐ \circ – role chain (composition): RoS
- ☐ $*$ – reflexive-transitive closure⁴: R^*
- ☐ id – concept identity: $id(C)$
- ☐ **Forbidden** – complex roles⁵ in number restrictions⁶

Role axioms (RBox):

- ☒ \mathcal{S} – Role transitivity: $\text{Trans}(R)$
- ☒ \mathcal{H} – Role hierarchy: $R \sqsubseteq S$
- ☐ \mathcal{R} – Complex role inclusions: $RoS \sqsubseteq R, RoS \sqsubseteq S$
- ☐ \mathcal{S} – some additional features

[Reset](#)

You have selected the Description Logic: **SHOIN**

Complexity of reasoning problems⁷

Reasoning problem	Complexity ⁸	Comments and references
Concept satisfiability	NExpTime-complete	<ul style="list-style-type: none"> • Hardness of even \mathcal{ALCFIO} is proved in [76, Corollary 4.13]. In that paper, the result is formulated for \mathcal{ALCFIO}, but only number restrictions of the form $(\leq 1R)$ are used in the proof. • A different proof of the NExpTime-hardness for \mathcal{ALCFIO} is given in [54] (even with 1 nominal, and role inverses not used in number restrictions). • Upper bound for \mathcal{SHOIQ} is proved in [77, Corollary 6.31] with numbers coded in unary (for binary coding, the upper bound remains an open problem for all logics in between $\mathcal{ALC/O}$ and \mathcal{SHOIQ}). • Important: in number restrictions, only <i>simple</i> roles (i.e. which are neither transitive nor have a transitive subroles) are allowed; otherwise we gain undecidability even in \mathcal{SHO}; see [46]. • Remark: recently [47] it was observed that, in many cases, one can use transitive roles in number restrictions – and still have a decidable logic! So the above notion of a <i>simple</i> role could be substantially extended.
ABox consistency	NExpTime-complete	By reduction to concept satisfiability problem in presence of nominals shown in [69, Theorem 3.7].



List of reasoners

Reasoner	Institution	Download	Publication
BaseVisor	VISIology, Inc.	Download	Core publication
BUNDLE	University of Ferrara	Download	Core publication
CEL	Technische Universität Dresden	Download	Core publication
Chainsaw	The University of Manchester	Download	Core publication
Clipper	Vienna University of Technology	Download	Core publication
DBOWL	University of Malaga	Download	Core publication
DeLorean	Not given	Download	Core publication

DieTEL	Wright State University	Download	Core publication
DRAOn	University of Paris 8, IUT of Montreuil	Download	Core publication
DrieW	Vienna University of Technology	Download	Core publication
ELepHant	Not given	Download	Core publication
ELK	University of Ulm, Germany	Download	Core publication
ELOG	Not given	Download	Core publication
FaCT++	The University of Manchester	Download	Core publication
fuzzyDL	ISTI - CNR	Download	Core publication

HermiT	University of Oxford	Download	Core publication
joel	Technische Universität Dresden	Download	Core publication
JFact	The University of Manchester	Download	Core publication
Konclude	University of Ulm, derivo GmbH	Download	Core publication
LIFR	Centre for Research and Technology Hellas (CERTH)	Download	Core publication
Mastro	Sapienza University of Rome	Download	Core publication
MORé	University of Oxford	Download	Core publication
ontop	Free University of Bozen-Bolzano	Download	Core publication



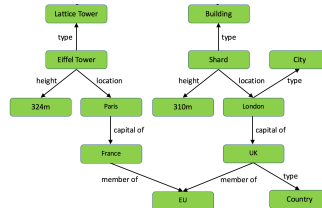
Standardised KR language

- RDF provides a graphical data model
- OWL provides a DL-based ontology language

OWL
ontology:

$\text{LatticeTower}(x) \rightarrow \text{ArchStruct}(x)$
 $\text{Building}(x) \rightarrow \text{ArchStruct}(x)$
 $\text{location}(x, y) \wedge \text{capitalOf}(y, z) \rightarrow \text{location}(x, z)$

RDF data:





Standardised KR language

- RDF provides a graphical data model
- OWL provides a DL-based ontology language

Developed as part of **W3C's Semantic Web** project

“A new form of Web content that is meaningful to computers will unleash a revolution of new possibilities” (!)



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Standardised KR language

- RDF provides a graphical data model
- OWL provides a DL-based ontology language

Developed as part of **W3C's Semantic Web** project

Now **widely used** in science,
healthcare and Industry

Often referred to as
"**semantic technology**"



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Based on powerful but still **decidable** DL (**SROIQ**)

Three “profiles” based on **tractable subsets**

- **QL**: based on the DL-Lite description logic
- **EL**: based on the EL description logic
- **RL**: based on the DL fragment of Datalog (aka DLP)

Different **algorithmic techniques**

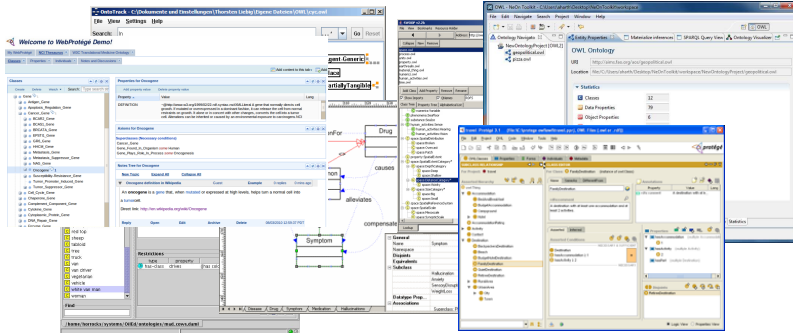
- (**Hyper-**) **Tableau** for full language
- **Query rewriting** for QL
- **Consequence-based** for EL
- **Materialisation** for RL

Highly **optimised implementations**

- Several of which have been developed here at Oxford



Tools:





Hermi **FaCT++** ORACLE'
Reasoners:
Racer



Applications: Question Answering



what country is the shard located in?



Google

what country is the shard located in?

All Maps News Images Shopping More Settings Tools

About 6,400,000 results (0.81 seconds)

The Shard / Country



United Kingdom

People also search for

View 10+ more



England London Great Britain Scotland United States of America Wales Ireland

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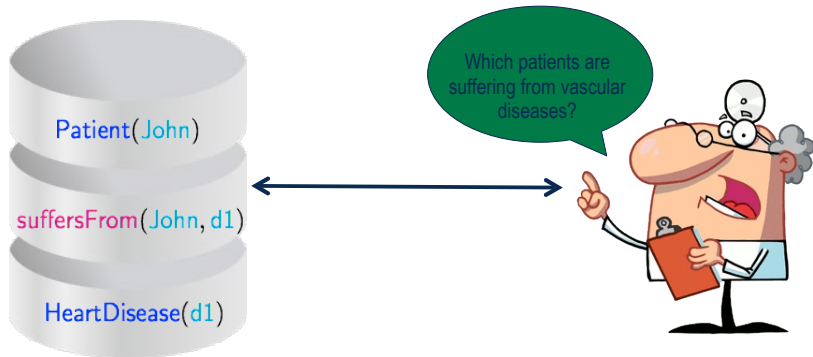
SNOMED is a **huge** medical ontology

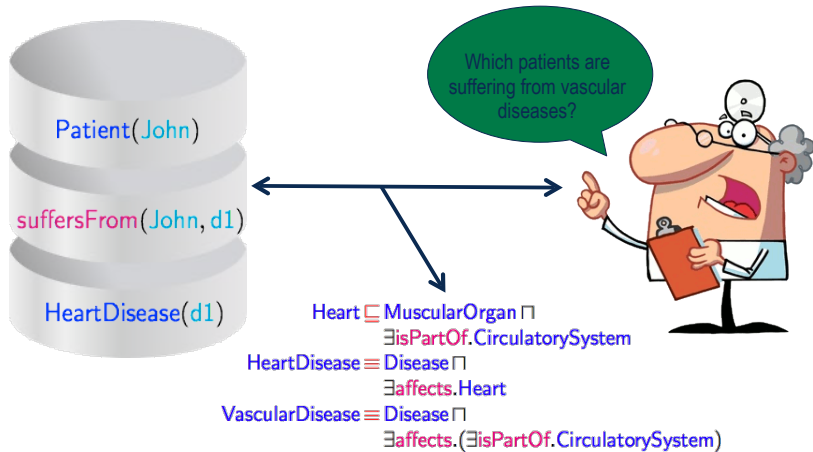
- More than 500,000 terms!

Why SNOMED? Let's ask Healthcare experts!

- "We need a clinical ontology that is **universal**, so any term I use is the same as every other colleague around the country"
- "SNOMED is the **glue** that binds the clinical community together and is the platform for all clinically relevant information"

Used to annotate patient records in **more than 20 countries**, including UK, USA, New Zealand, ...





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The end?

Extensions

- Arithmetic functions and aggregation
- Reasoning about time
- Data streams

Algorithms

- Consequence-based reasoning
- Hybrid rewriting/materialisation

Optimisation and implementation

- Incremental reasoning
- Query planning
- HPC, including large-scale and distributed architectures

Tools and applications

Course Structure

Logics for KRR

- Propositional and First Order Logic
- Ontological modelling
- The role of reasoning

Horn logics and Datalog

- Expressivity and formal properties
- Reasoning

Description Logic

- Motivation and foundations
- Model theory
- Tableau reasoning
- Reasoning with data
- Lightweight description logics
- Ontology based data access

Course Structure

Description Logic -v- Datalog

- Expressiveness and decidability
- Combining DL and Datalog
- Other decidable fragments of FOL

Ontology Languages and Semantic Technologies

- RDF & OWL
- SROIQ
- Non-DL features
- Profiles
- Tools and Reasoners

Nonmonotonic Reasoning

- Limitations of FOL
- Closed World Assumption and negation as failure
- Stable model semantics

Reading List

Primary Text

- An Introduction to Description Logic. Franz Baader, Ian Horrocks, Carsten Lutz, Uli Sattler

Supplementary Texts

- Handbook of Knowledge Representation. Frank van Harmelen, Vladimir Lifschitz and Bruce Porter (Eds). Foundations of Artificial Intelligence, 2008.
- Foundations of Semantic Web Technologies. Chapman & Hall/ CRC Textbooks in Computing. Pascal Hitzler, Markus Kroetsch, and Sebastian Rudolph, 2009.