Pushing the Boundaries of Tractable Multiperspective Reasoning

A Deduction Calculus for Standpoint $\mathscr{EL}$+

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International Center for Computational Logic

TECHNISCHE UNIVERSITÄT DRESDEN

European Research Council
Established by the European Commission

Motivation

Multiperspective Reasoning

Motivation: Knowledge Integration

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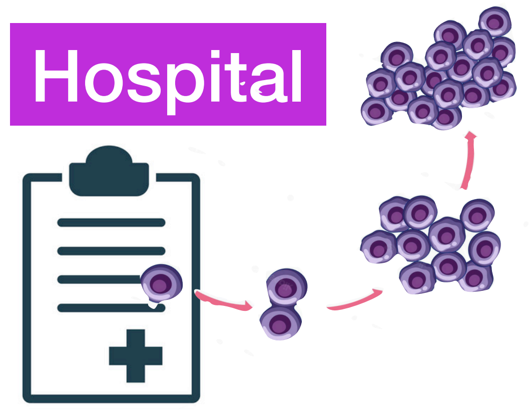
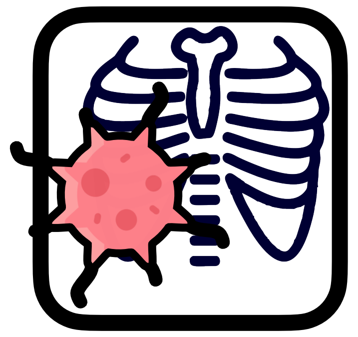


WIKIDATA

yago select knowledge

Private Lab

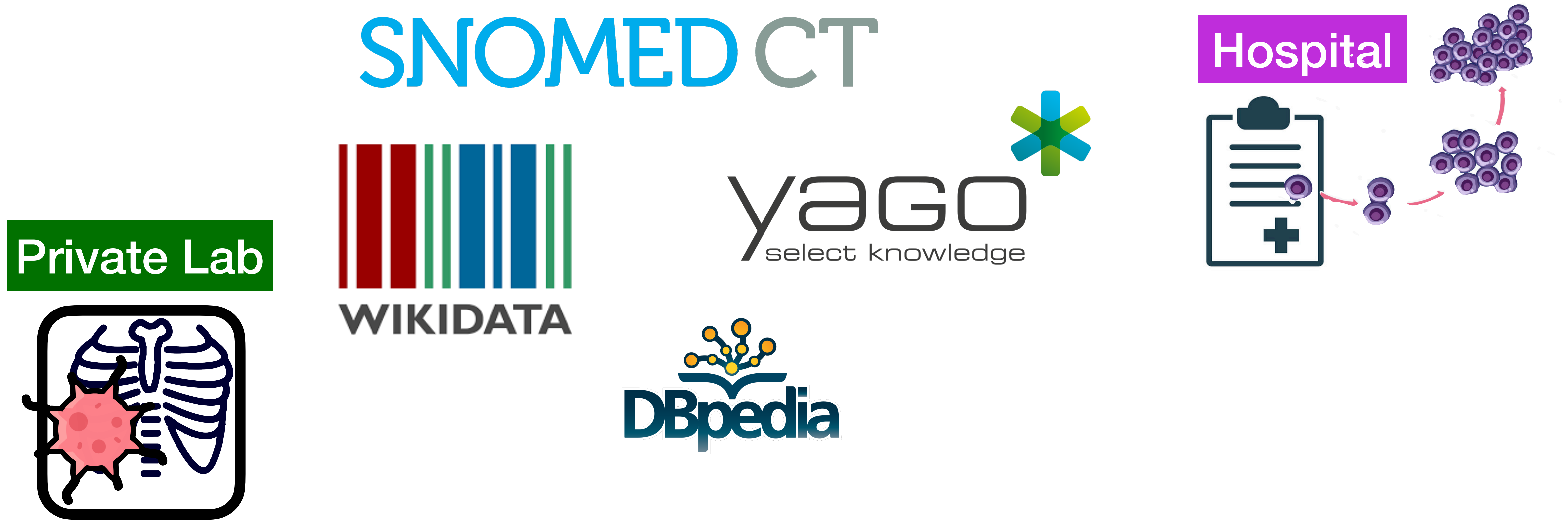


DBpedia

Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available



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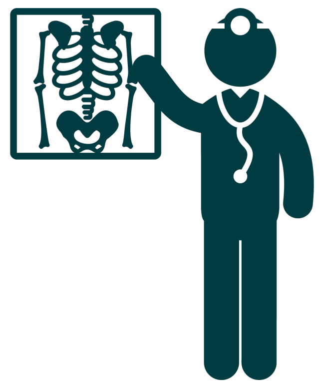

Tumor(t)
patientPart(p, t)

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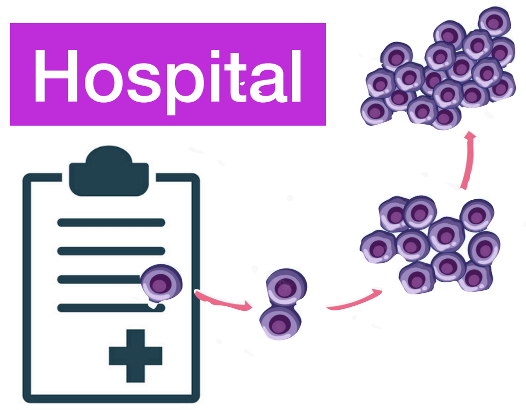

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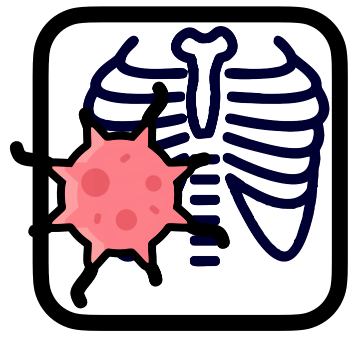

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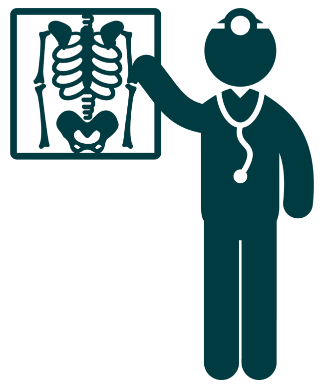

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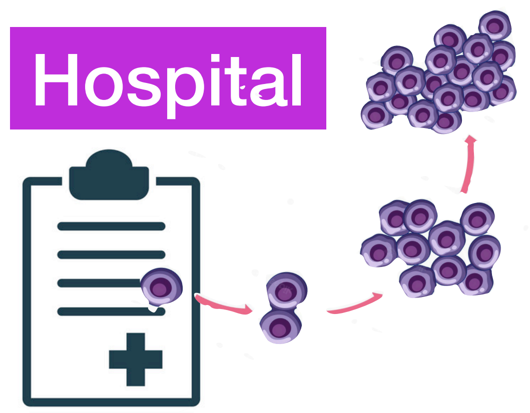

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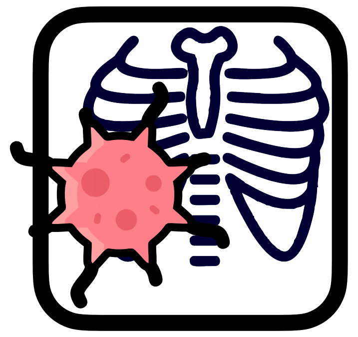

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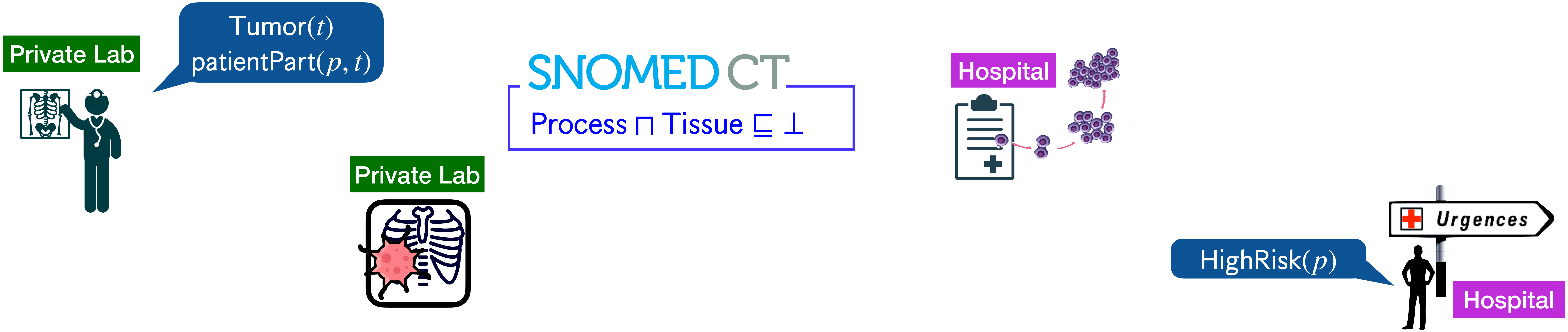

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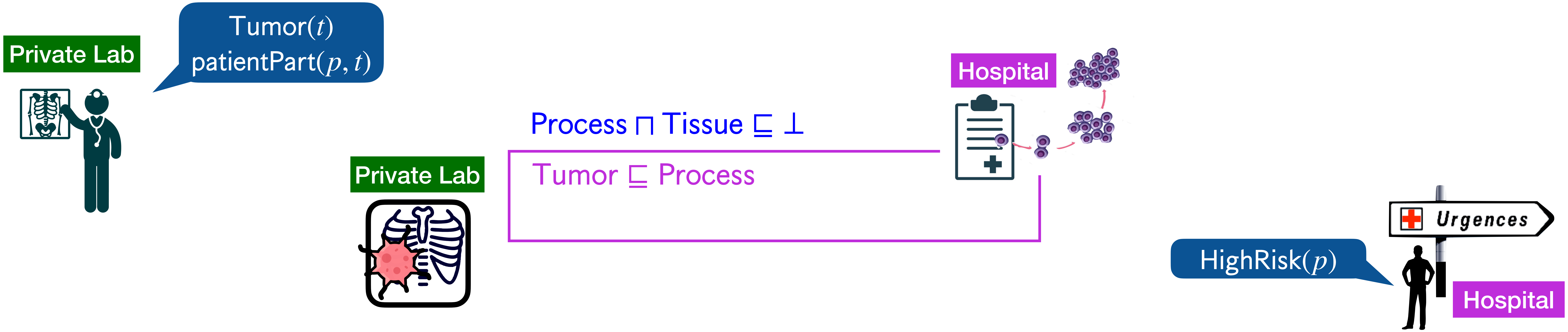

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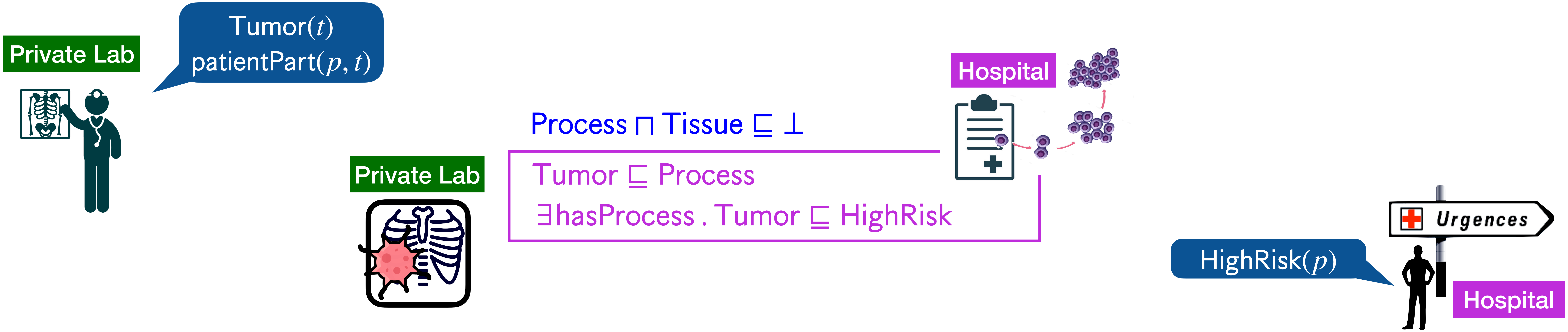

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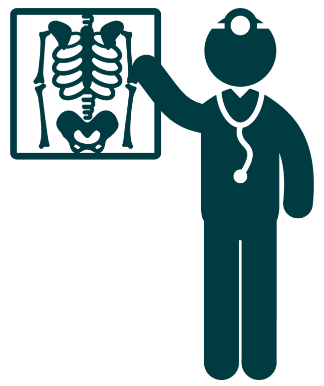

$\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot$

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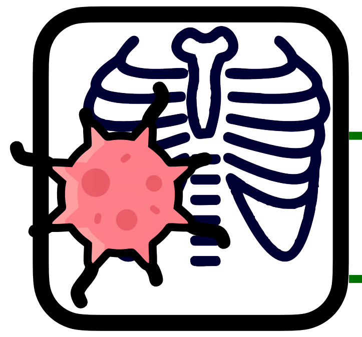

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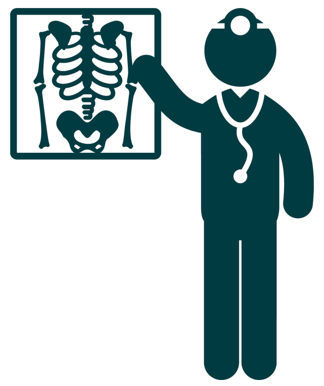

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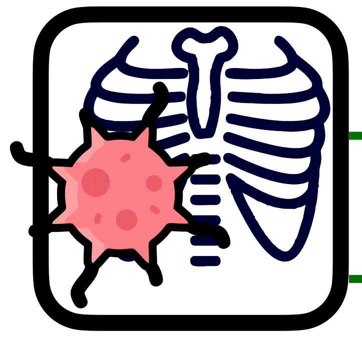

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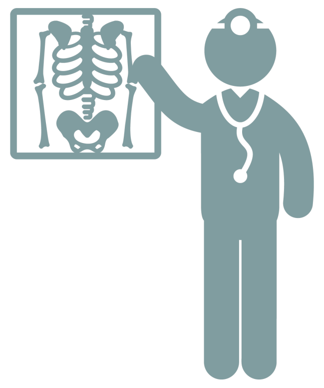

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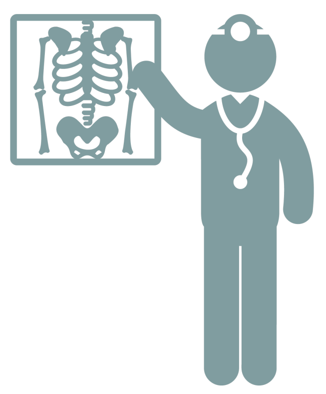

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Knowledge Fusion

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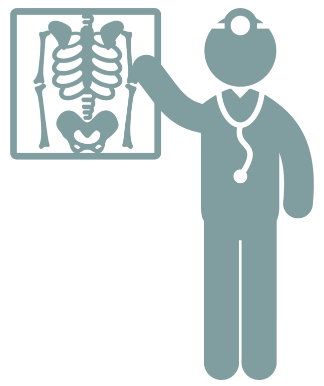

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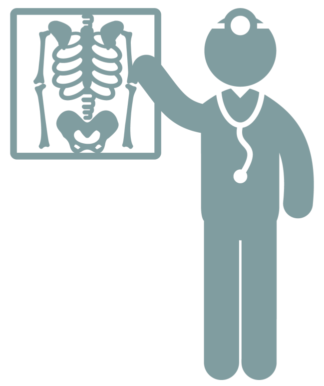

Knowledge Fusion

Process $\sqcap$ Tissue $\sqsubseteq \bot$

Tumor $\sqsubseteq$ Process

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Tumor $\sqsubseteq$ Tissue

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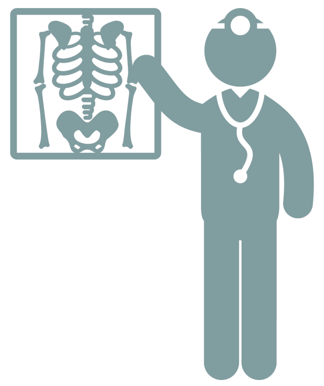

Knowledge Fusion

Process ⊓ Tissue ⊑ ⊥

Tumor ⊑ Process
∃hasProcess . Tumor ⊑ HighRisk

Tumor ⊑ Tissue

HighRisk(p)

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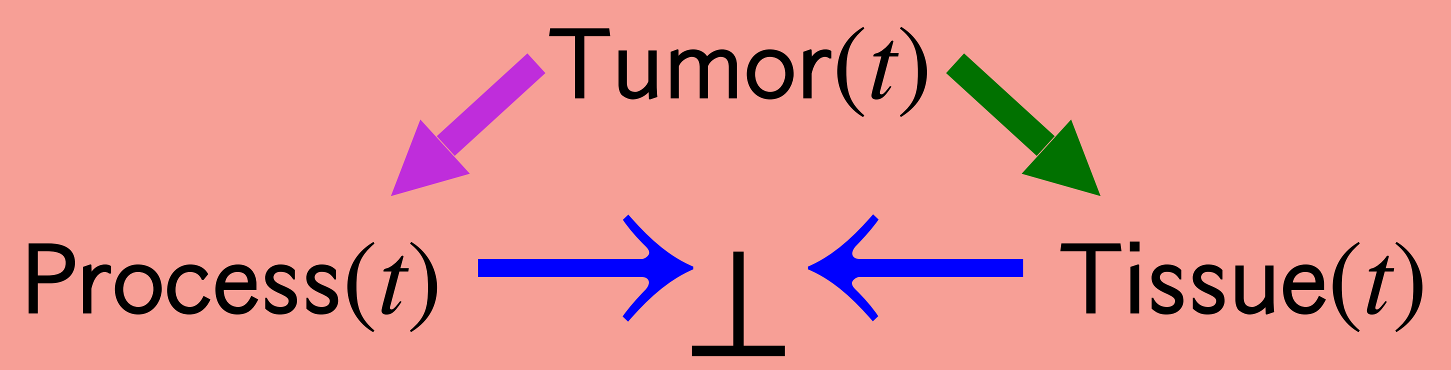

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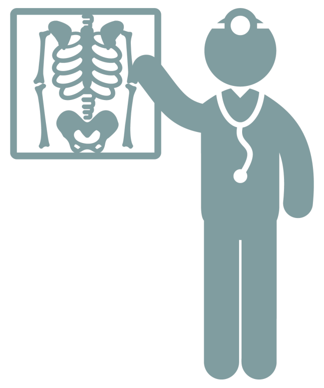

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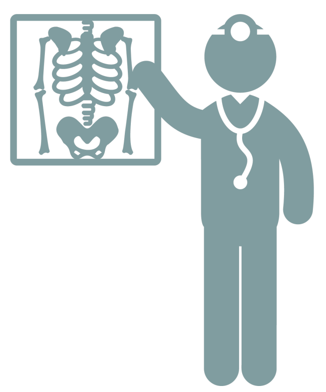
- $\mathsf{Tumor}(t) \rightarrow \mathsf{Process}(t)$
- $\mathsf{Tumor}(t) \rightarrow \mathsf{Tissue}(t)$
- $\mathsf{Process}(t) \rightarrow \bot \leftarrow \mathsf{Tissue}(t)$

Private Lab



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Challenge: combining diverse (potentially conflicting) sources without weakening them

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$\Box_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L [\text{Tumor}] \sqsubseteq \Box_L [\text{Tissue}]$

$\Diamond_H [\text{Tumor}] \sqsubseteq \Box_H [\text{Process}]$

$\Box_H [\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}]$

$\Box_e$ Unequivocal to e
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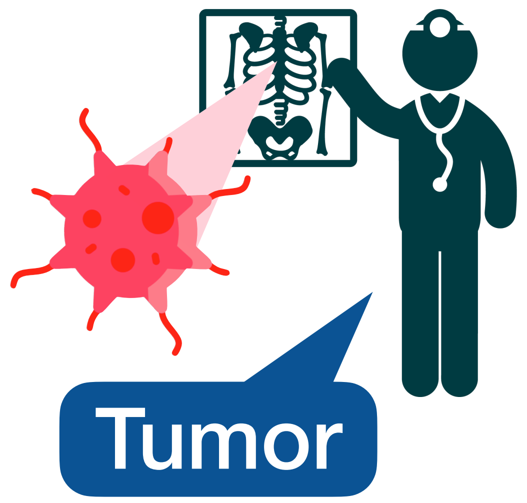

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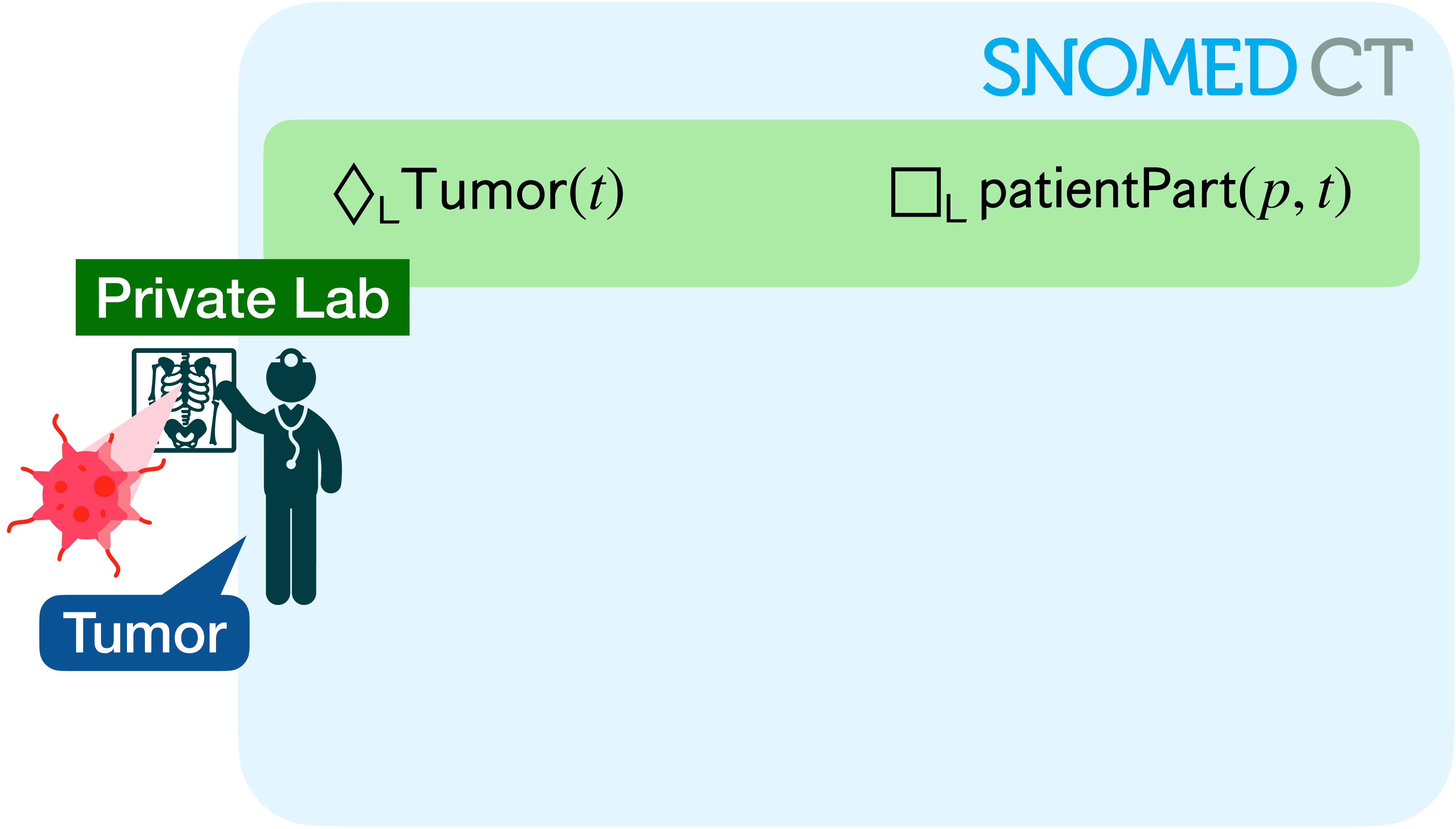

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SNOMED CT

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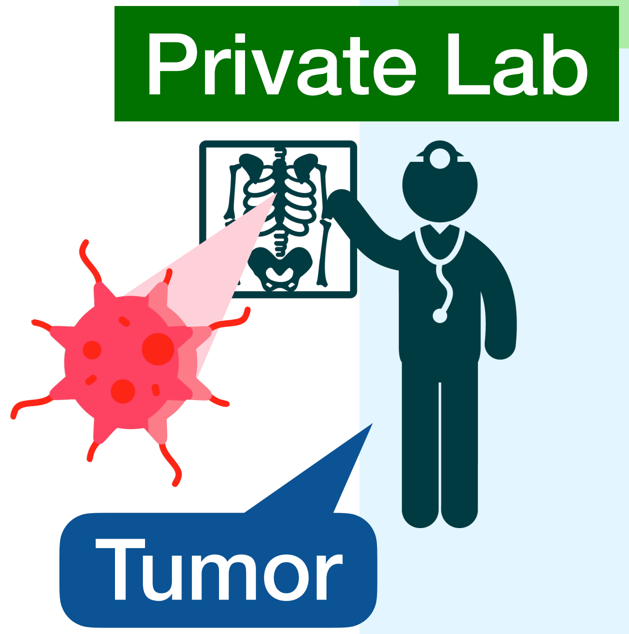

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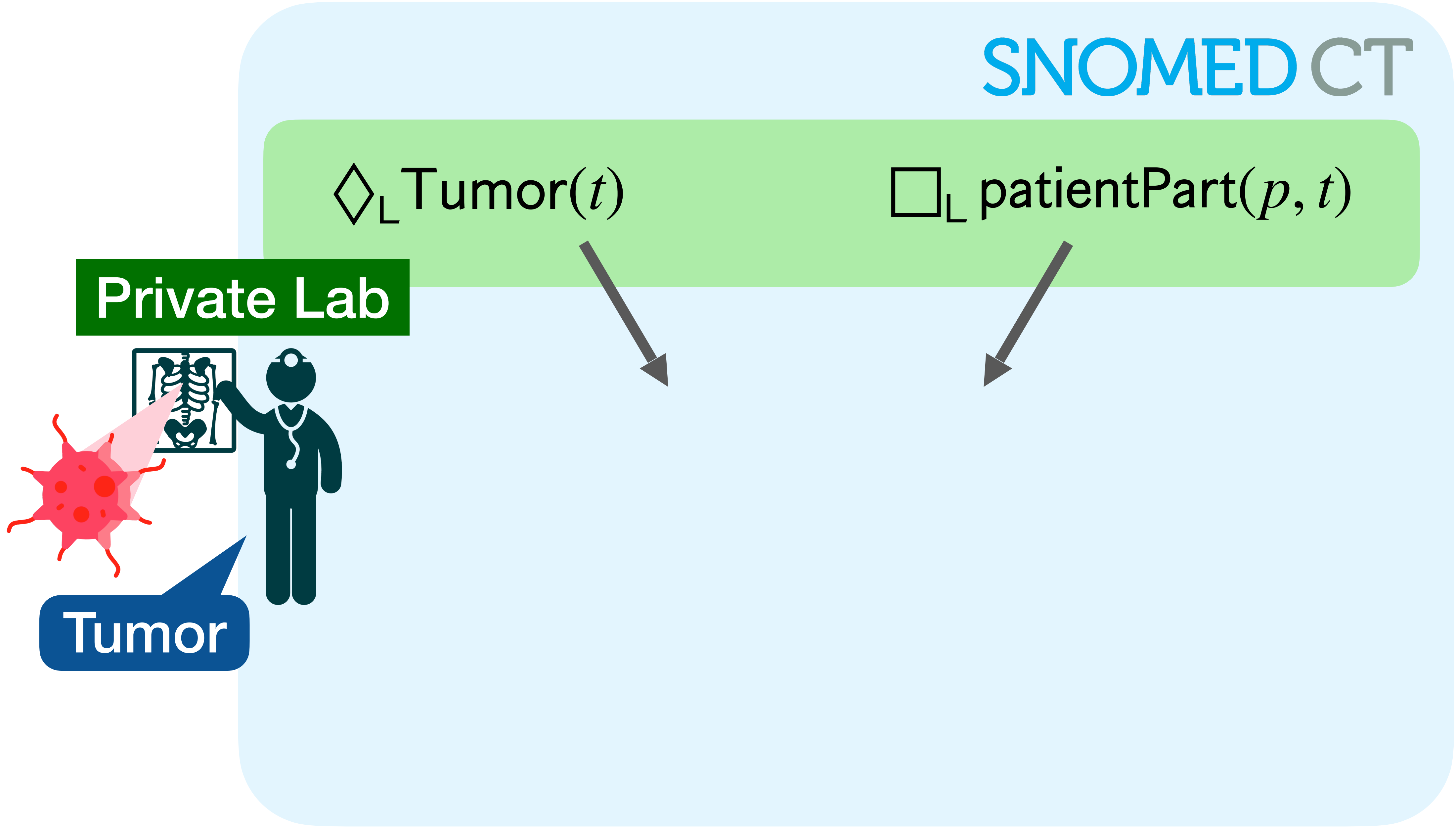

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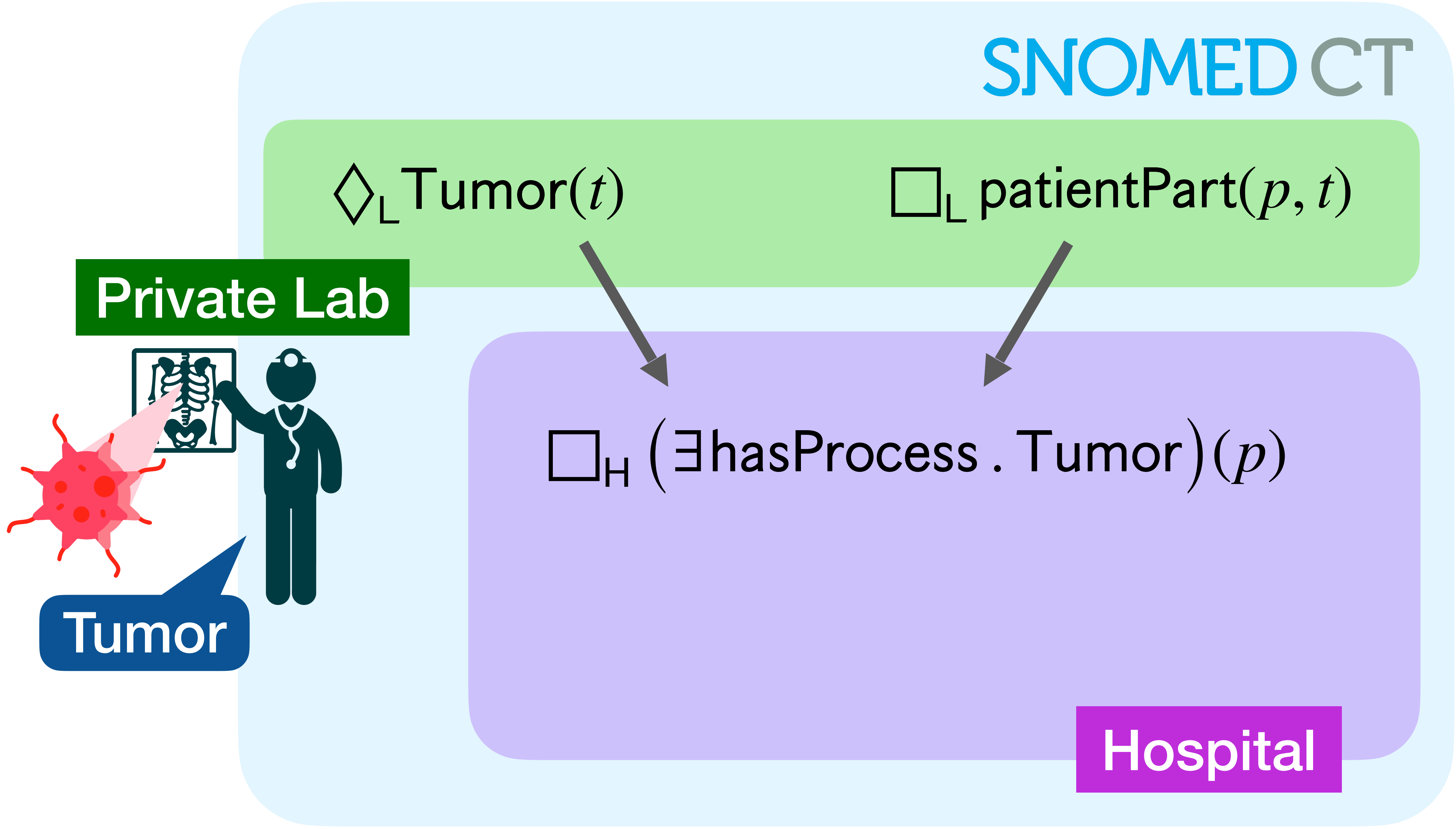

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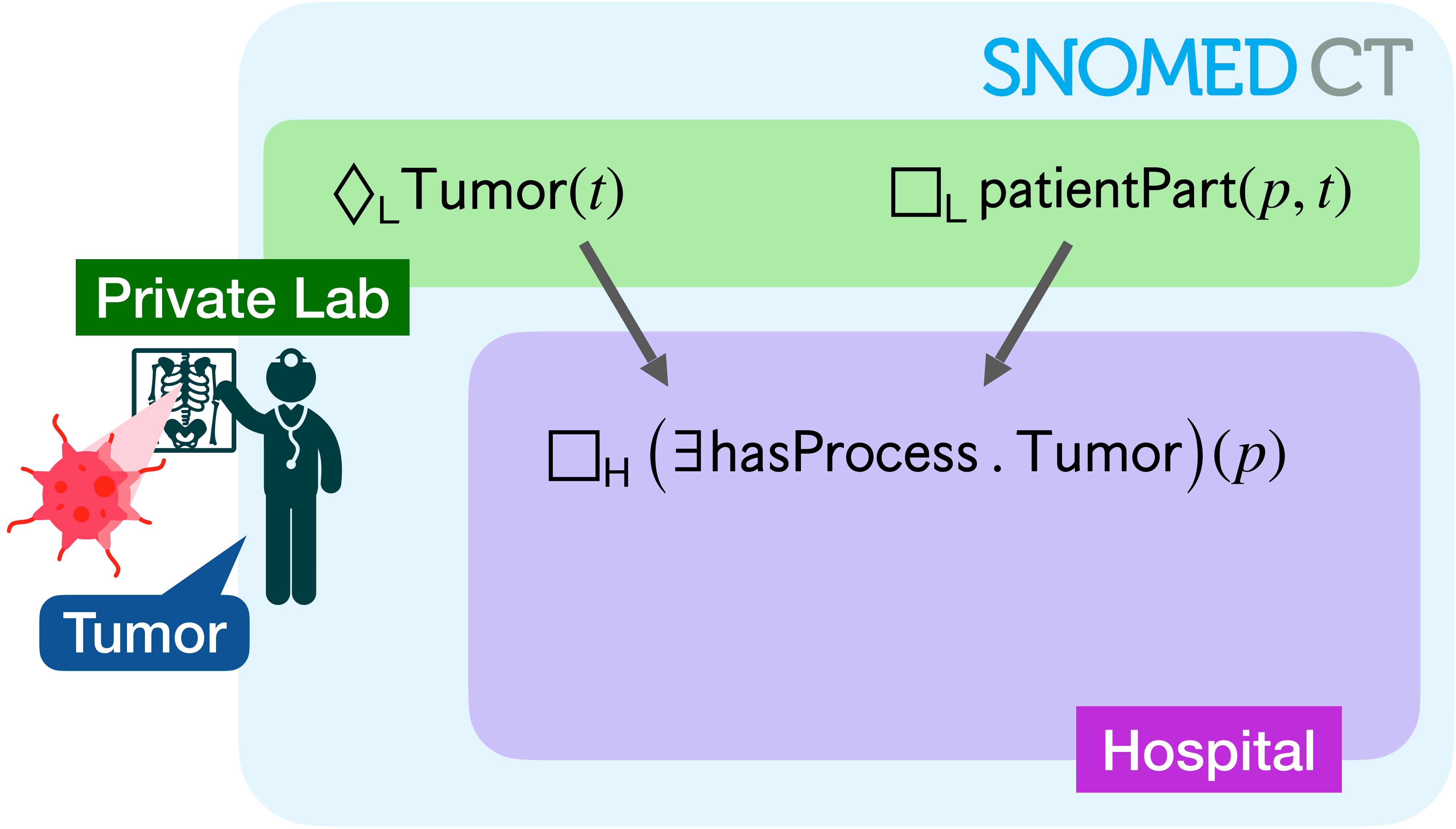

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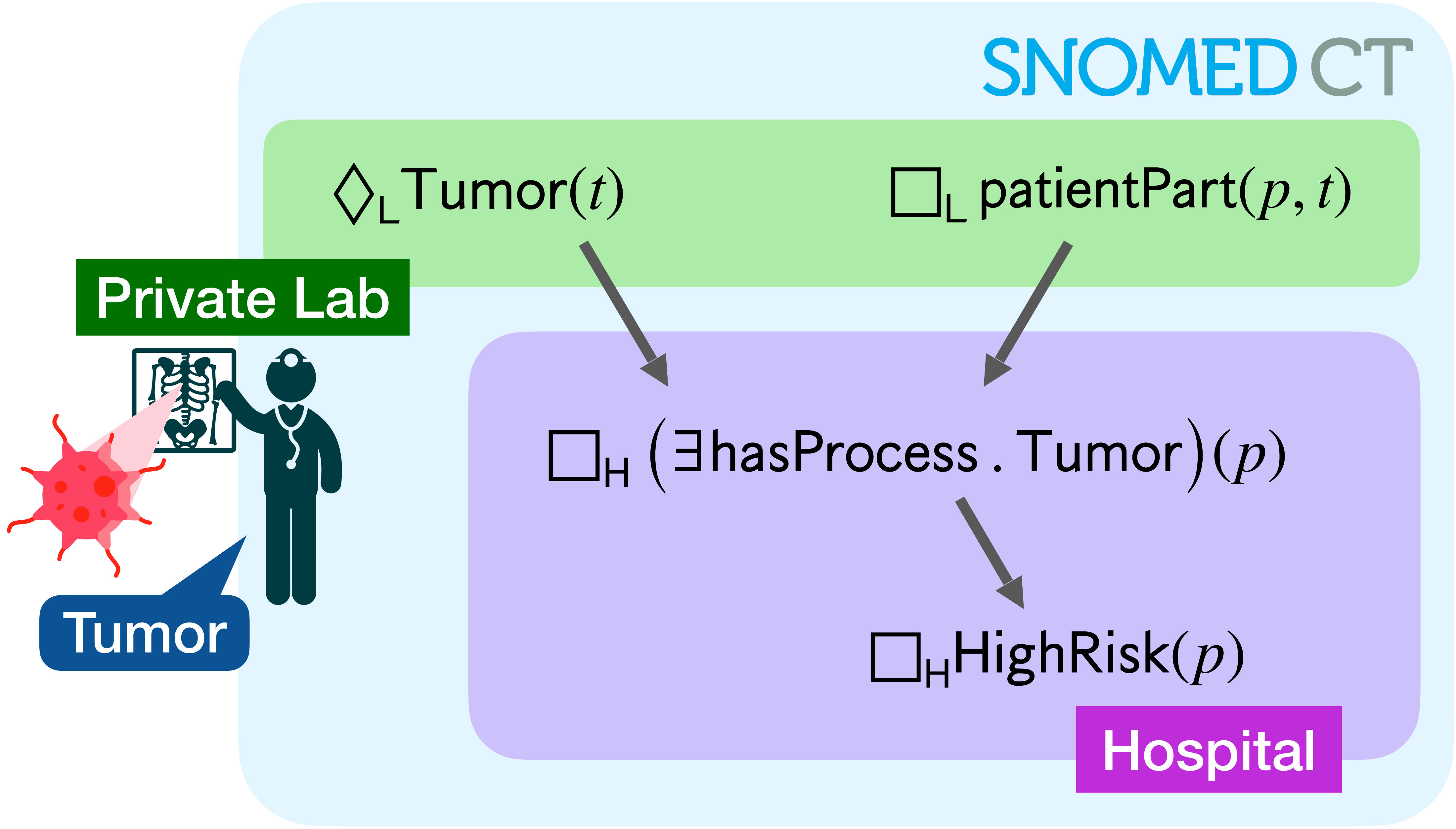
$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\Diamond_L [\exists \text{patientPart} . \text{Tumor}] \sqsubseteq \square_H [\exists \text{hasProcess} . \text{Tumor}]$



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\square_e$ Unequivocal to e
$\lozenge_e$ Conceivable to e

$\square_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

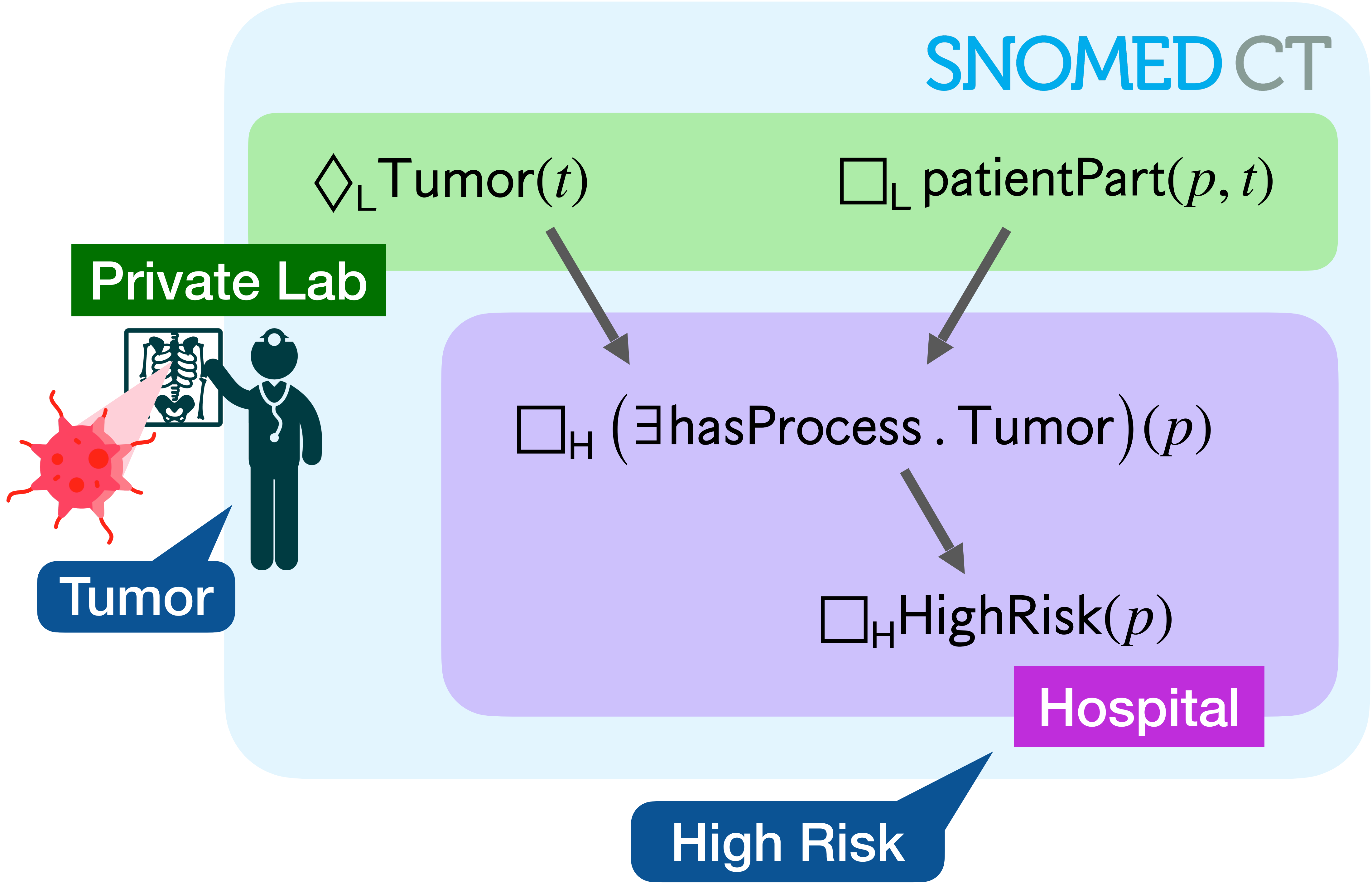
$\lozenge_L [\text{Tumor}] \sqsubseteq \square_L [\text{Tissue}]$

$\lozenge_H [\text{Tumor}] \sqsubseteq \square_H [\text{Process}]$

$\square_H [\exists \text{hasProcess}.\text{Tumor} \sqsubseteq \text{HighRisk}]$

$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\lozenge_L [\exists \text{patientPart}.\text{Tumor}] \sqsubseteq \square_H [\exists \text{hasProcess}.\text{Tumor}]$



Standpoint $\mathcal{EL}^+$

The description logic $\mathscr{EL}$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

The description logic $\mathscr{E}\mathscr{L}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process ⊓ Tissue

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_C$, $r \in N_R$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$

(Tumor $\sqsubseteq$ Tissue)

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$ $(\exists \text{patientPart}\,.\,\text{Tumor})(p)$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} . \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart} . \mathsf{Tumor})(p)$

Semantics:

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_C$, $r \in N_R$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_C, r \in N_R$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_C$, $r \in N_R$

Tissue

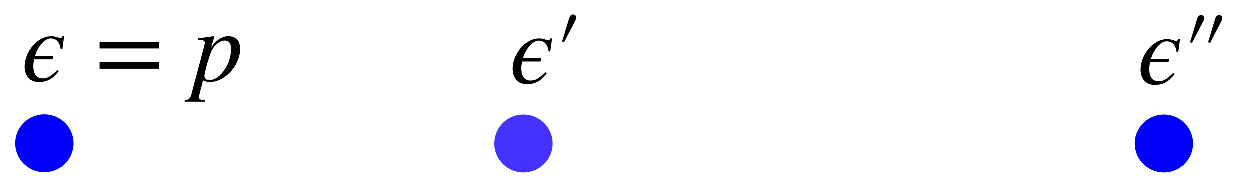
Process $\sqcap$ Tissue $\quad$ $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $\quad C(a), \quad r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue}) \qquad (\exists\text{patientPart}\,.\,\text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$

With $A \in N_C, r \in N_R$

Tissue

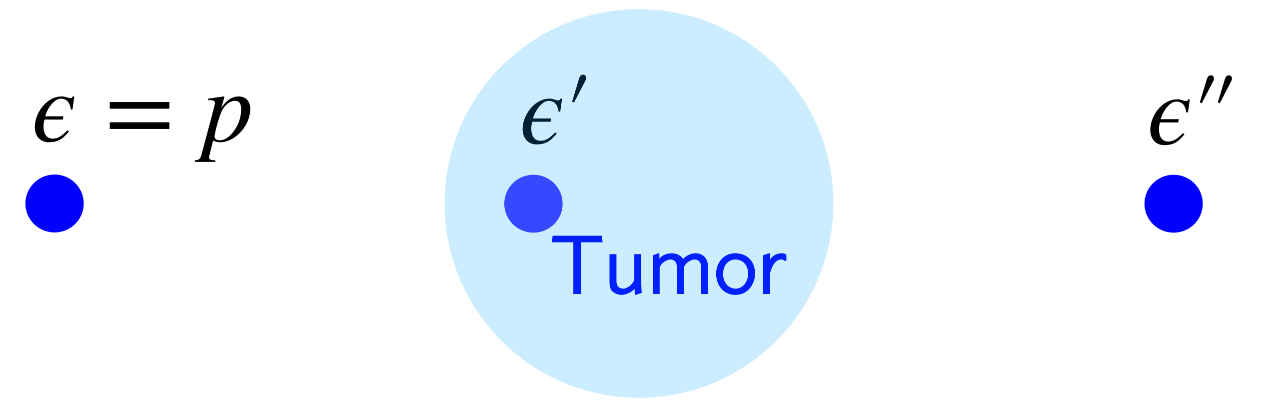
Process $\sqcap$ Tissue　　$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs　$C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue)　($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_C, r \in N_R$

Tissue

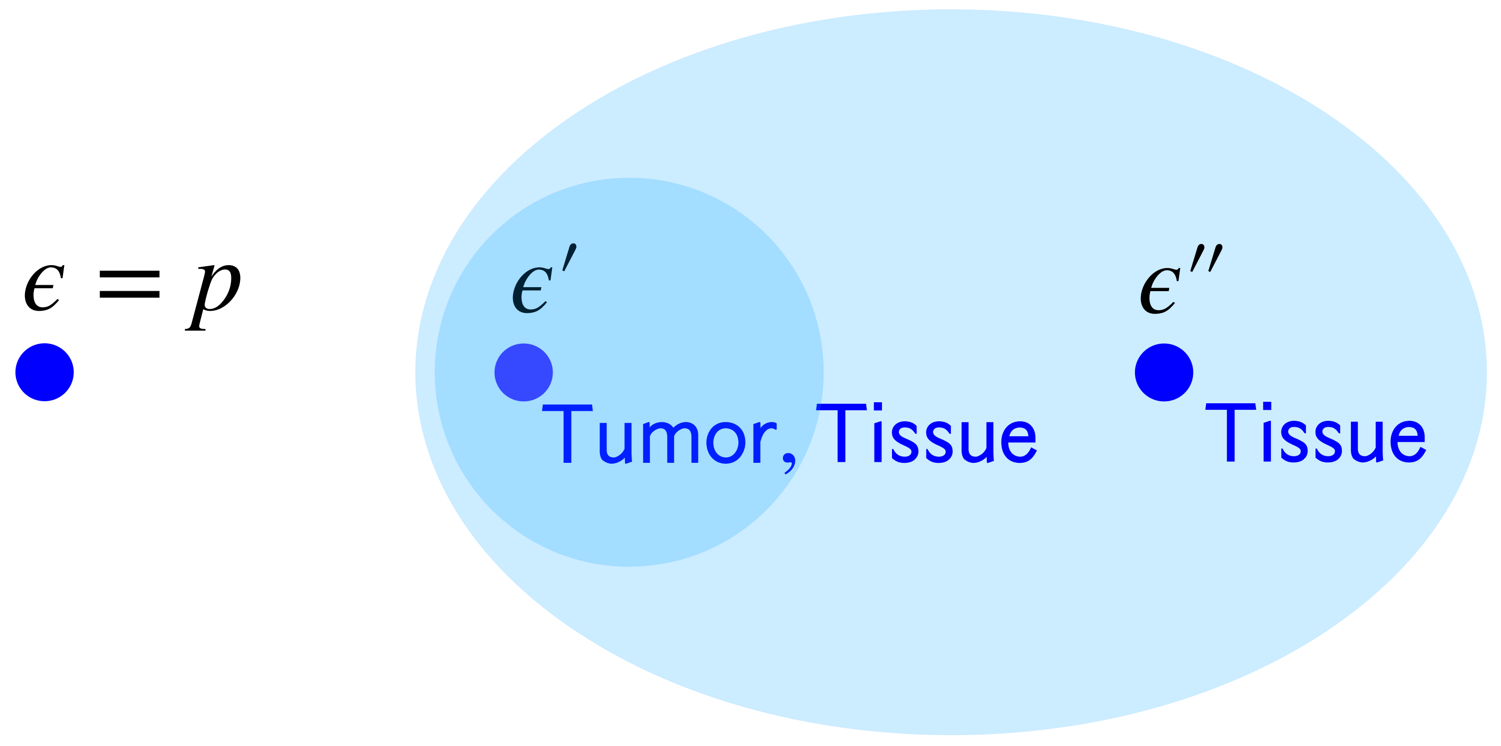
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

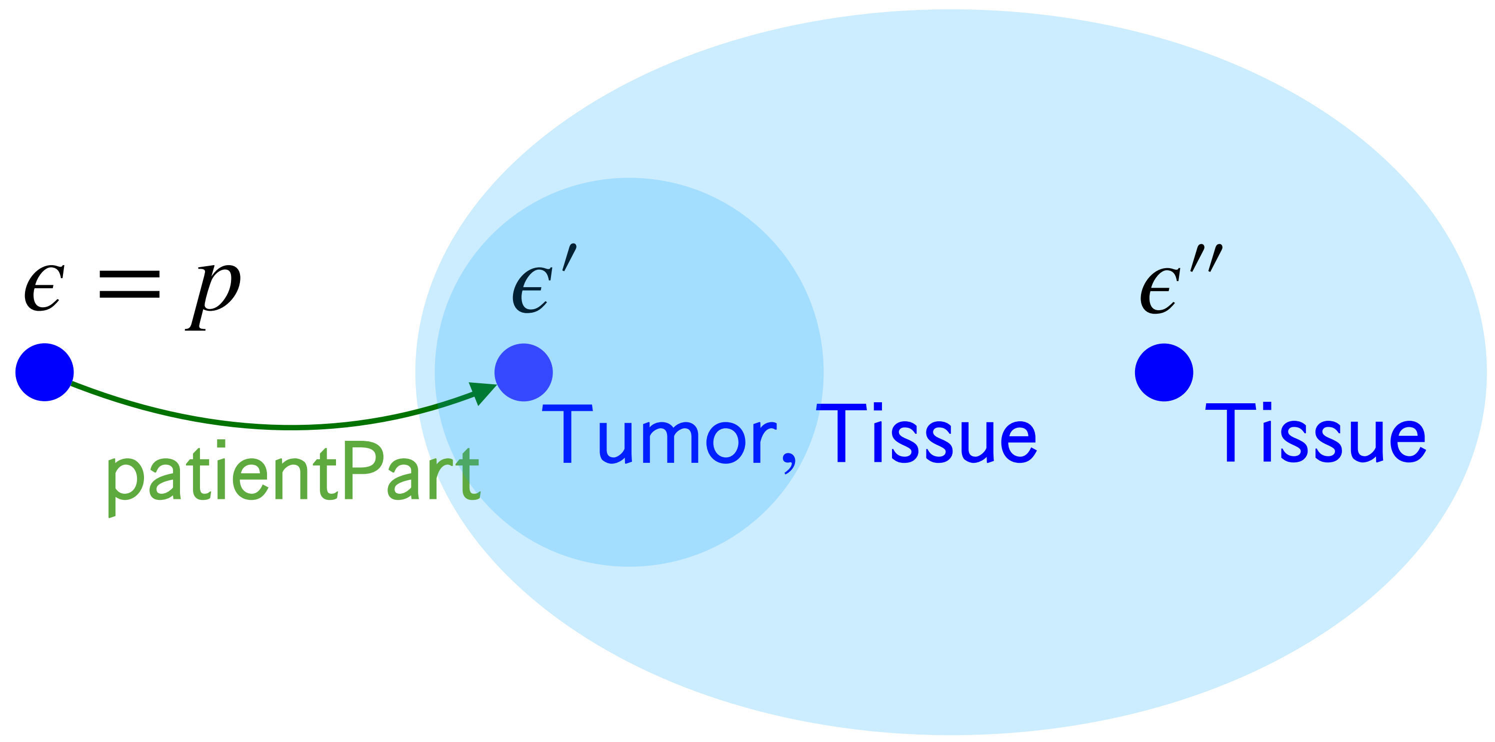
$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_C, r \in N_R$

Tissue

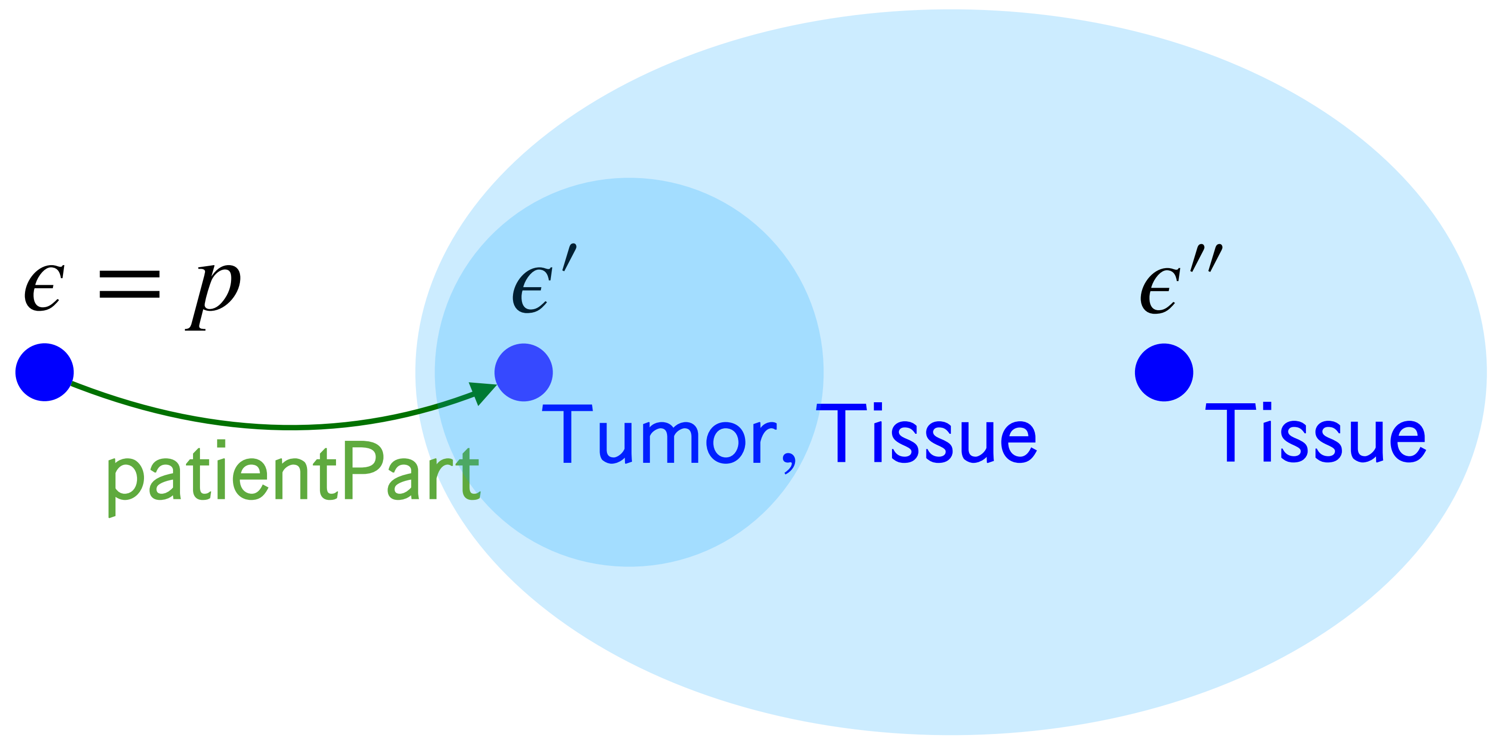
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_C$, $r \in N_R$

Tissue

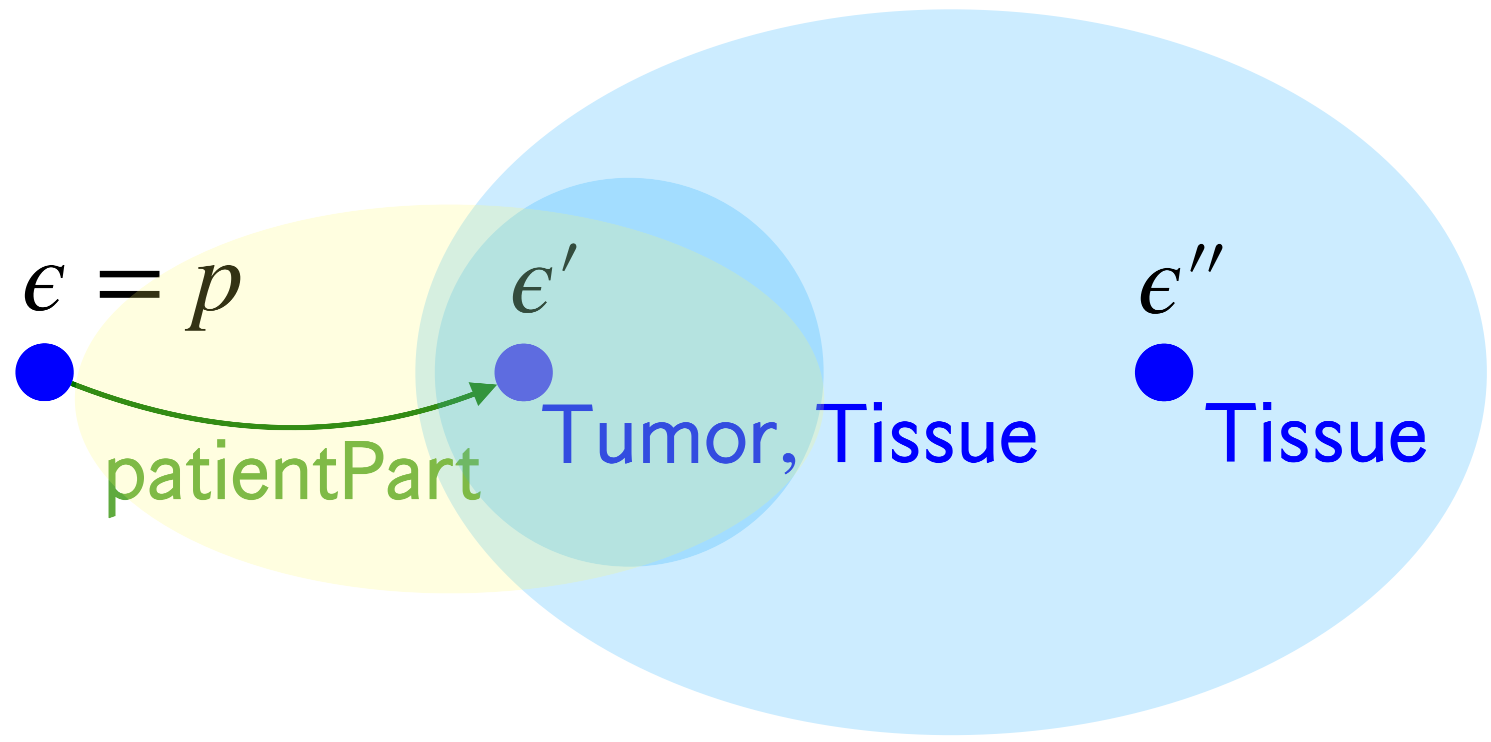
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$

The description logic $\mathscr{EL}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_C, r \in N_R$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a,b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . Self$

With $A \in N_C, r \in N_R$

Tissue
Process $\sqcap$ Tissue
$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

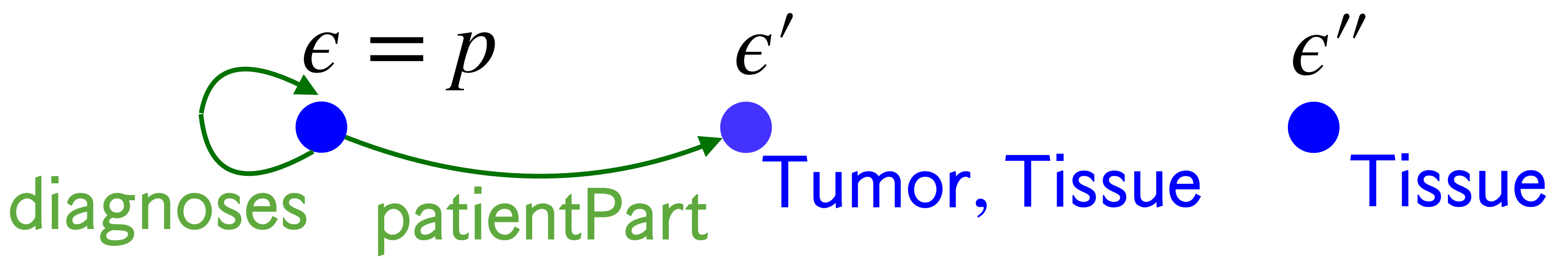
Tissue $\exists$diagnoses . Self
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue) ($\exists$patientPart . Tumor)(p)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathscr{EL}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . Self$

With $A \in N_C, r \in N_R$

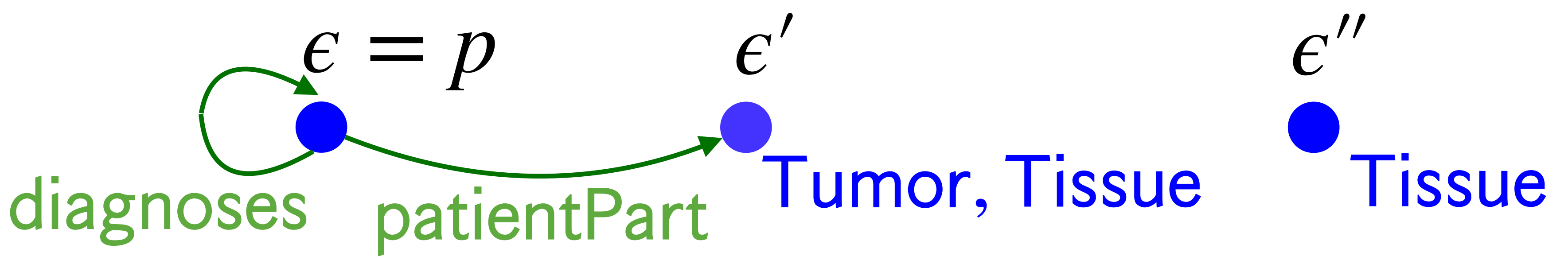
$\text{Tissue} \quad \exists \text{diagnoses} . \text{Self}$
$\text{Process} \sqcap \text{Tissue} \quad \exists \text{patientPart} . \text{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{EL}$+

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r.C \mid \exists r.\mathit{Self}$

With $A \in N_C, r \in N_R$

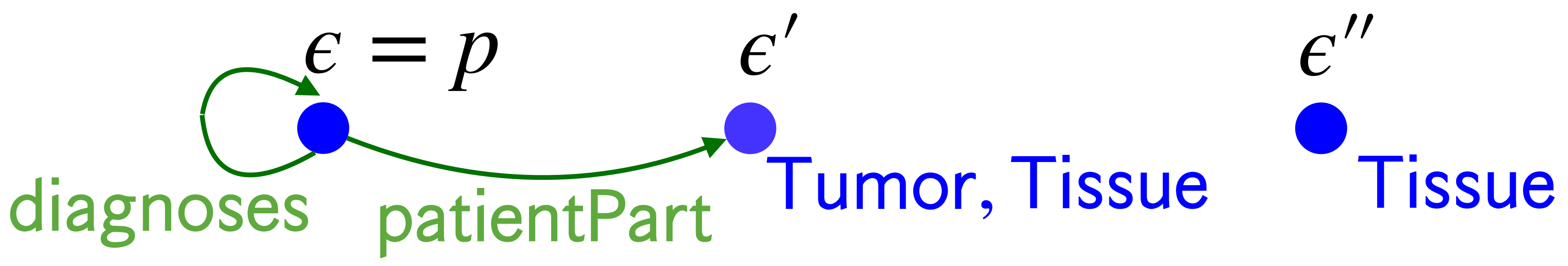
Tissue ∃diagnoses . Self
Process ⊓ Tissue ∃patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

(Tumor ⊑ Tissue) (patientPart ∘ hasPart ⊑ patientPart)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathcal{EL}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . \mathit{Self}$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

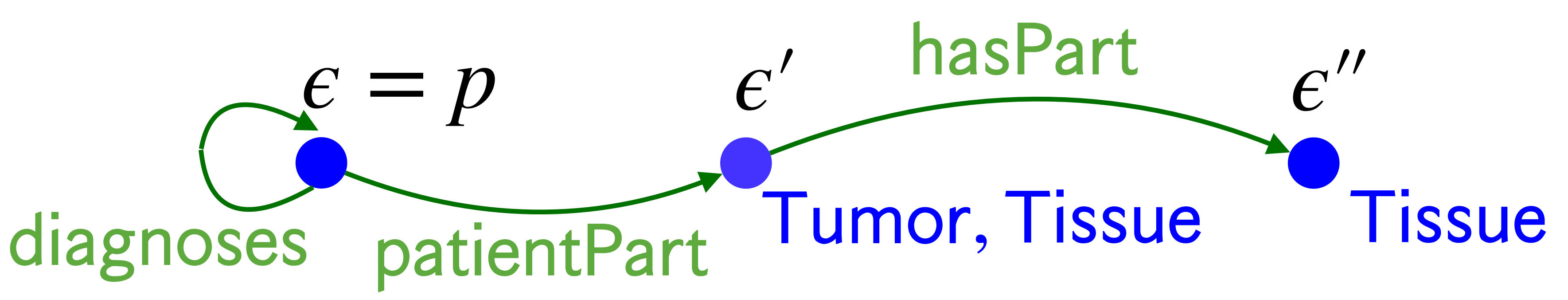
Tissue $\exists$diagnoses . Self
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

(Tumor $\sqsubseteq$ Tissue)   (patientPart $\circ$ hasPart $\sqsubseteq$ patientPart)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathcal{EL}$+

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . \mathit{Self}$

With $A \in N_C$, $r \in N_R$

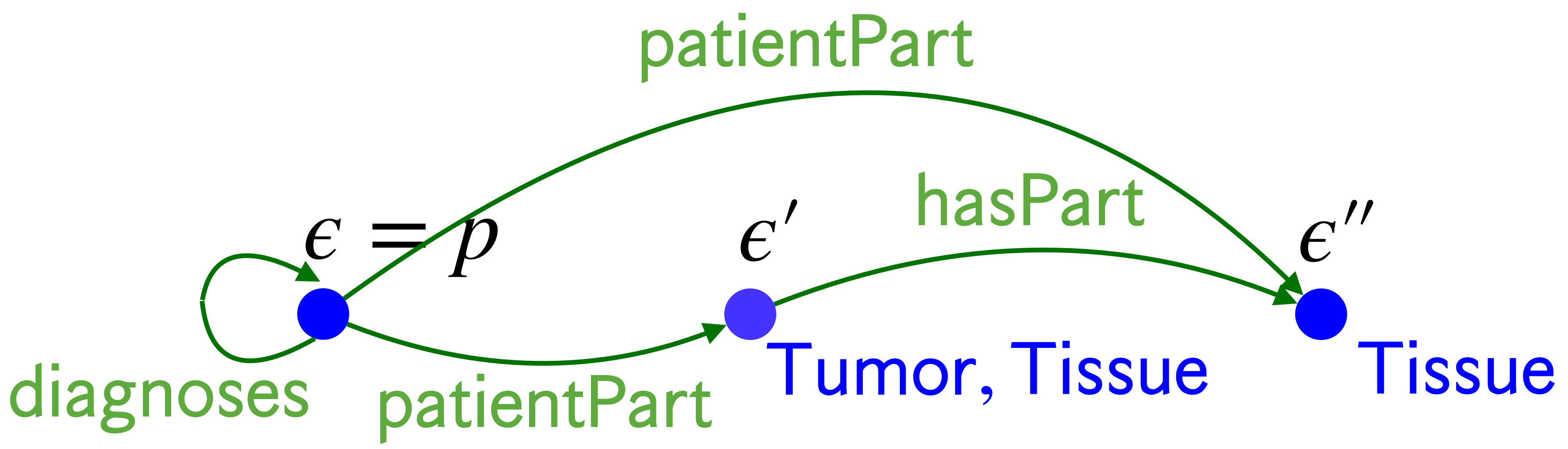
Tissue ∃diagnoses . Self
Process ⊓ Tissue ∃patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

(Tumor ⊑ Tissue) (patientPart ∘ hasPart ⊑ patientPart)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



Towards Standpoint-$\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self$$

With $A \in N_C$, $r \in N_R$

Tissue $\quad \exists \text{diagnoses} . \text{Self}$

Process $\sqcap$ Tissue $\quad \exists \text{patientPart} . \text{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue}) \quad (\exists \text{patientPart} . \text{Tumor})(p)$

Towards Standpoint-$\mathcal{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, Self$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue $\quad \exists \mathsf{diagnoses} \,.\, \mathsf{Self}$

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad \exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $\quad (\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_C, N_R, N_I, N_S \rangle$ of concept, role, individual and standpoint names, $* \in N_S$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . Self$$

With $A \in N_C, r \in N_R$

Tissue $\exists \mathsf{diagnoses} . \mathsf{Self}$

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} . \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

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Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}}, N_{\mathsf{S}} \rangle$ of concept, role, individual and standpoint names, $* \in N_{\mathsf{S}}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self \mid \odot_s C$$

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Tissue $\quad \exists \mathsf{diagnoses}\,.\,\mathsf{Self}$
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad \exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

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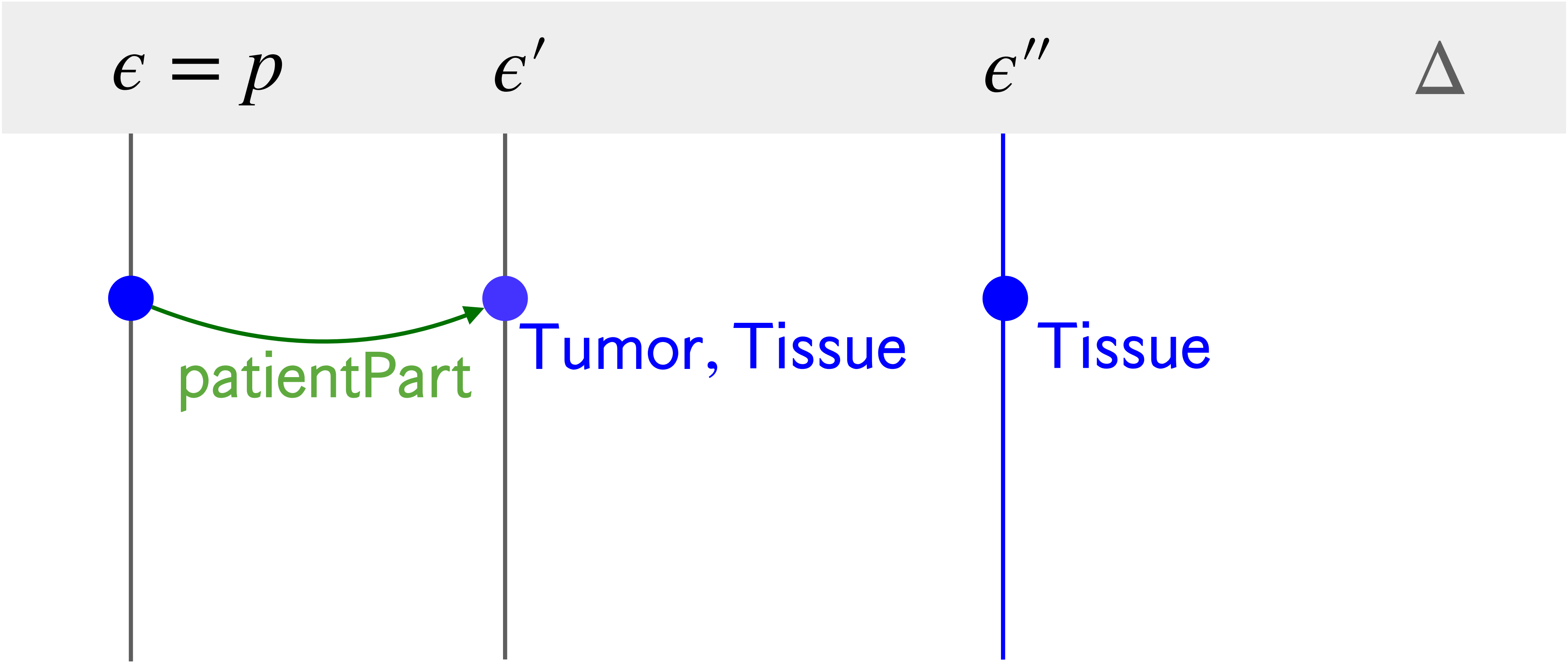

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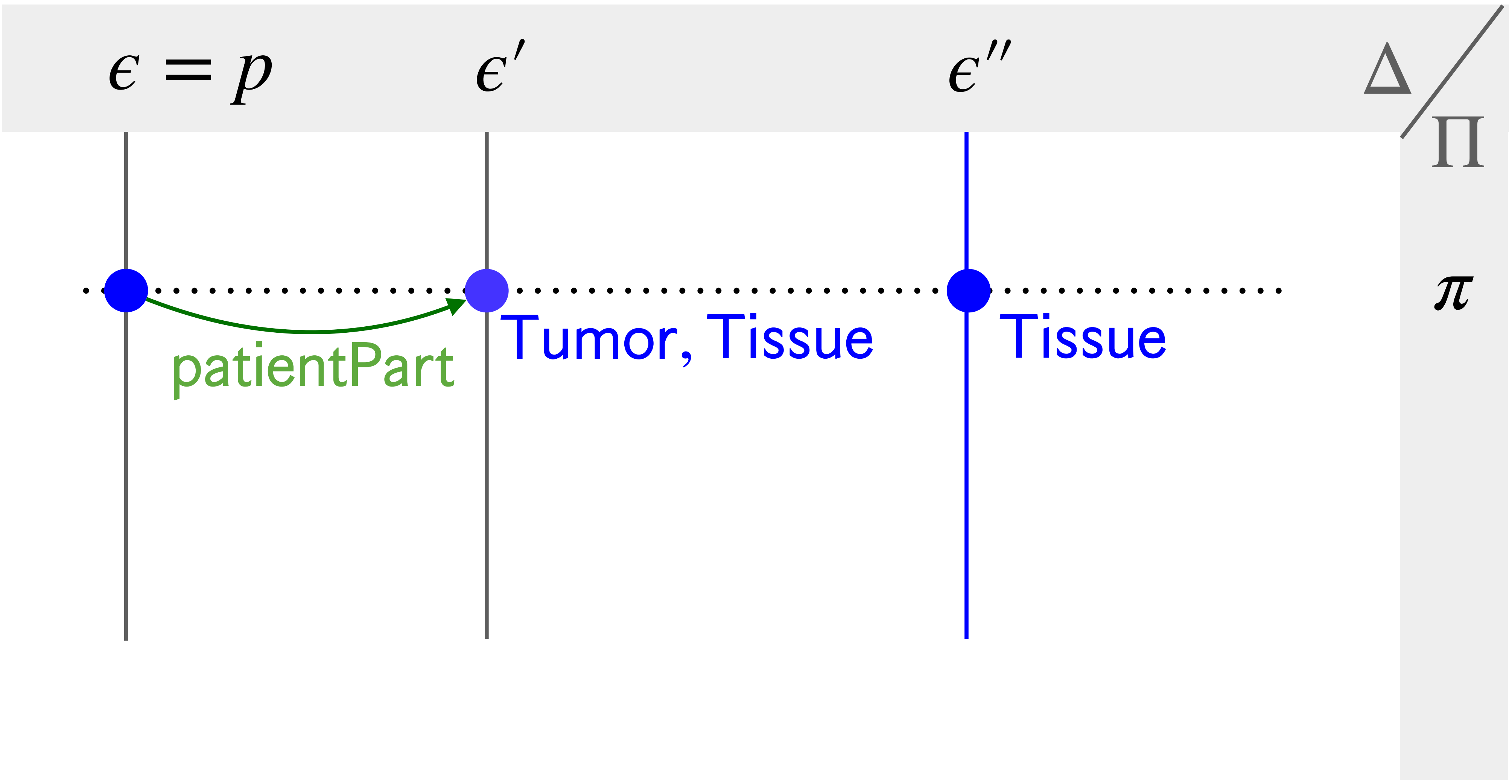

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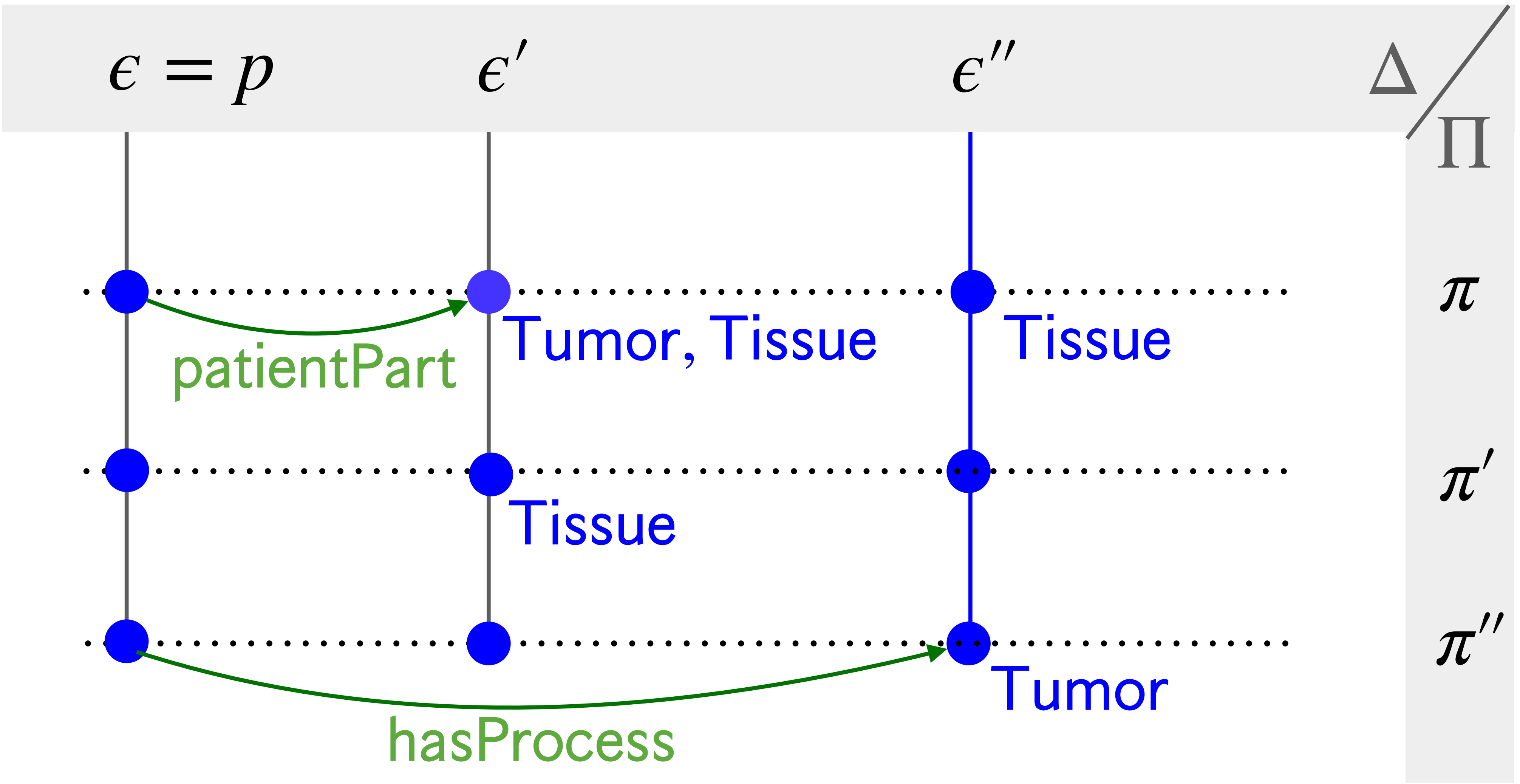

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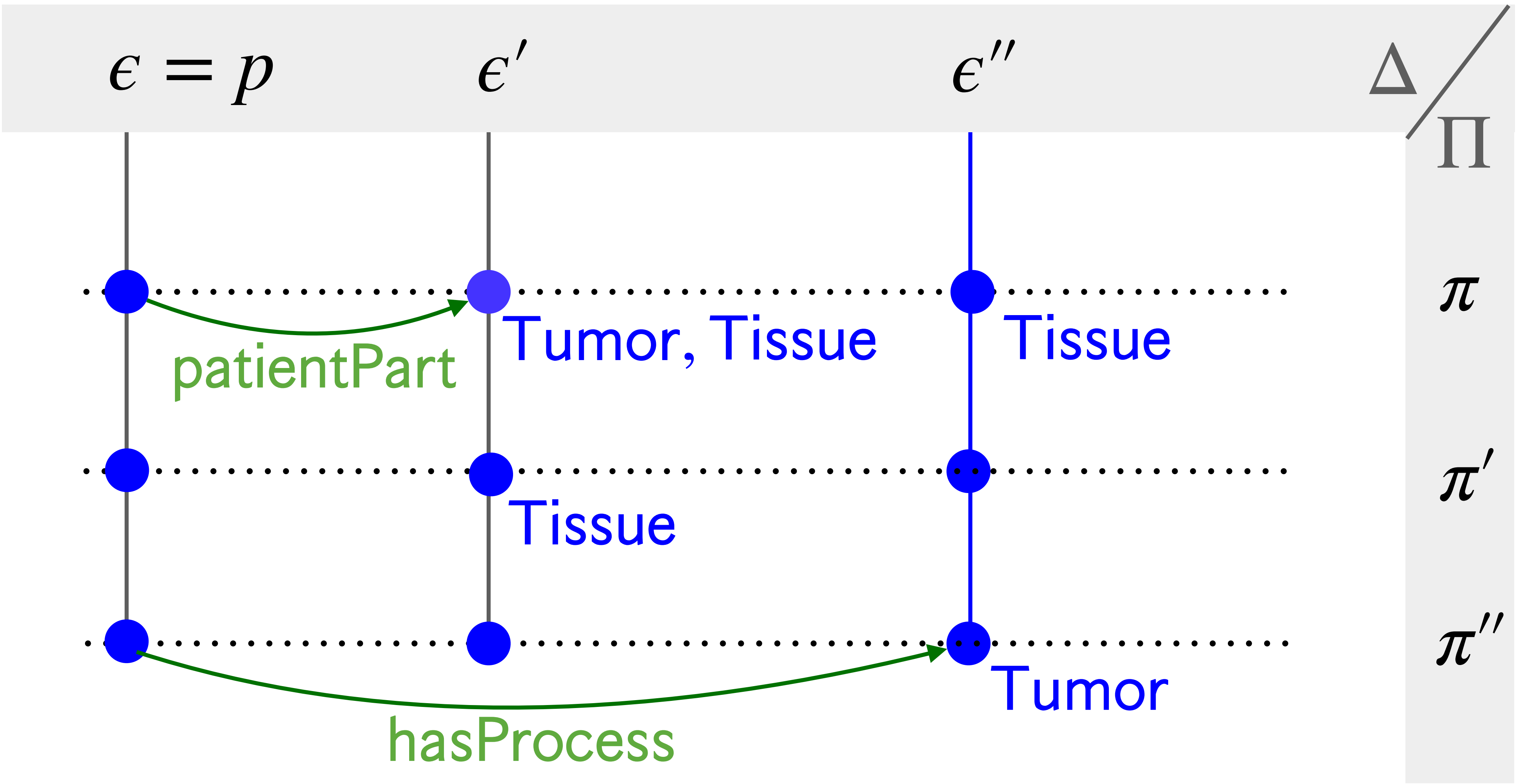

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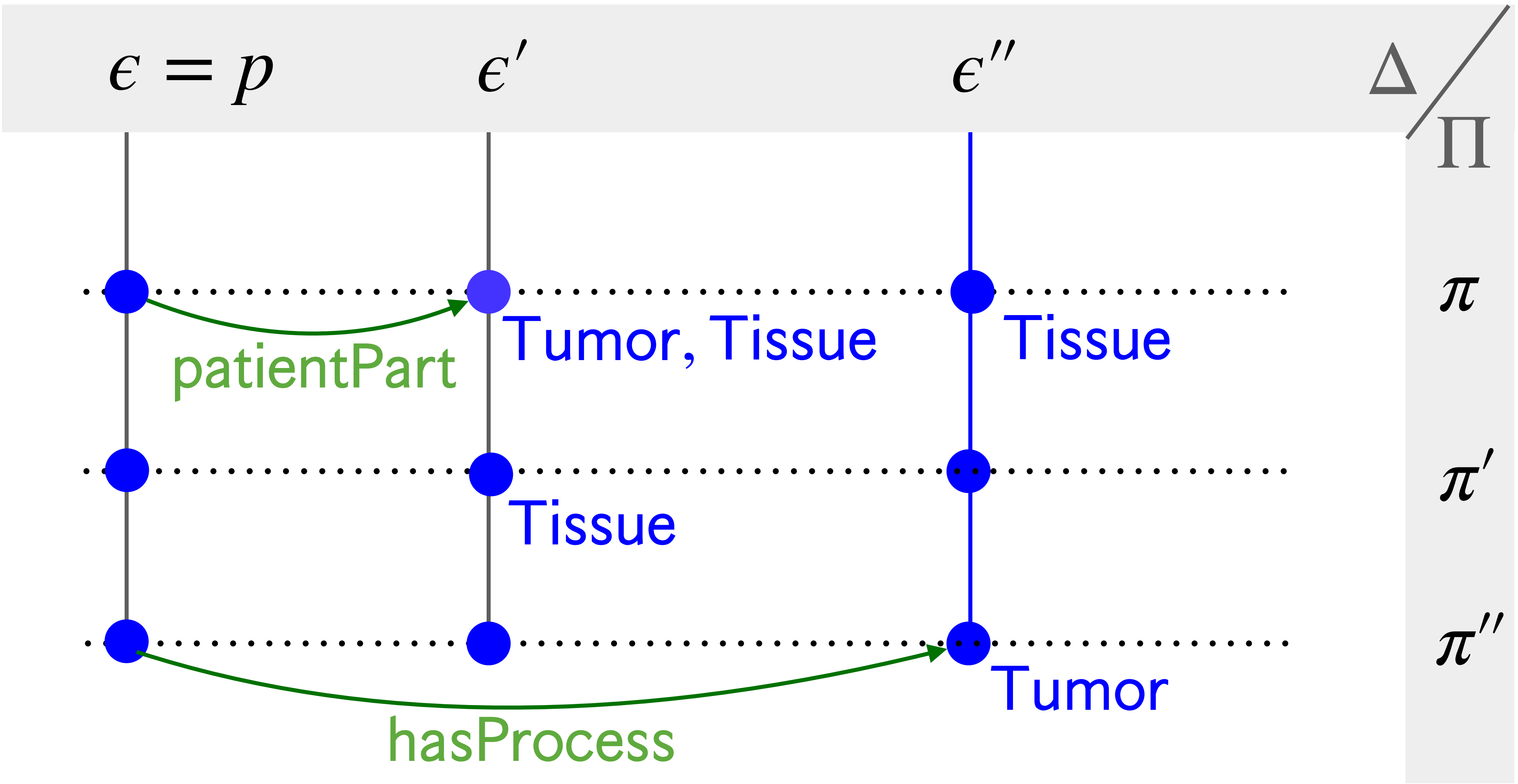

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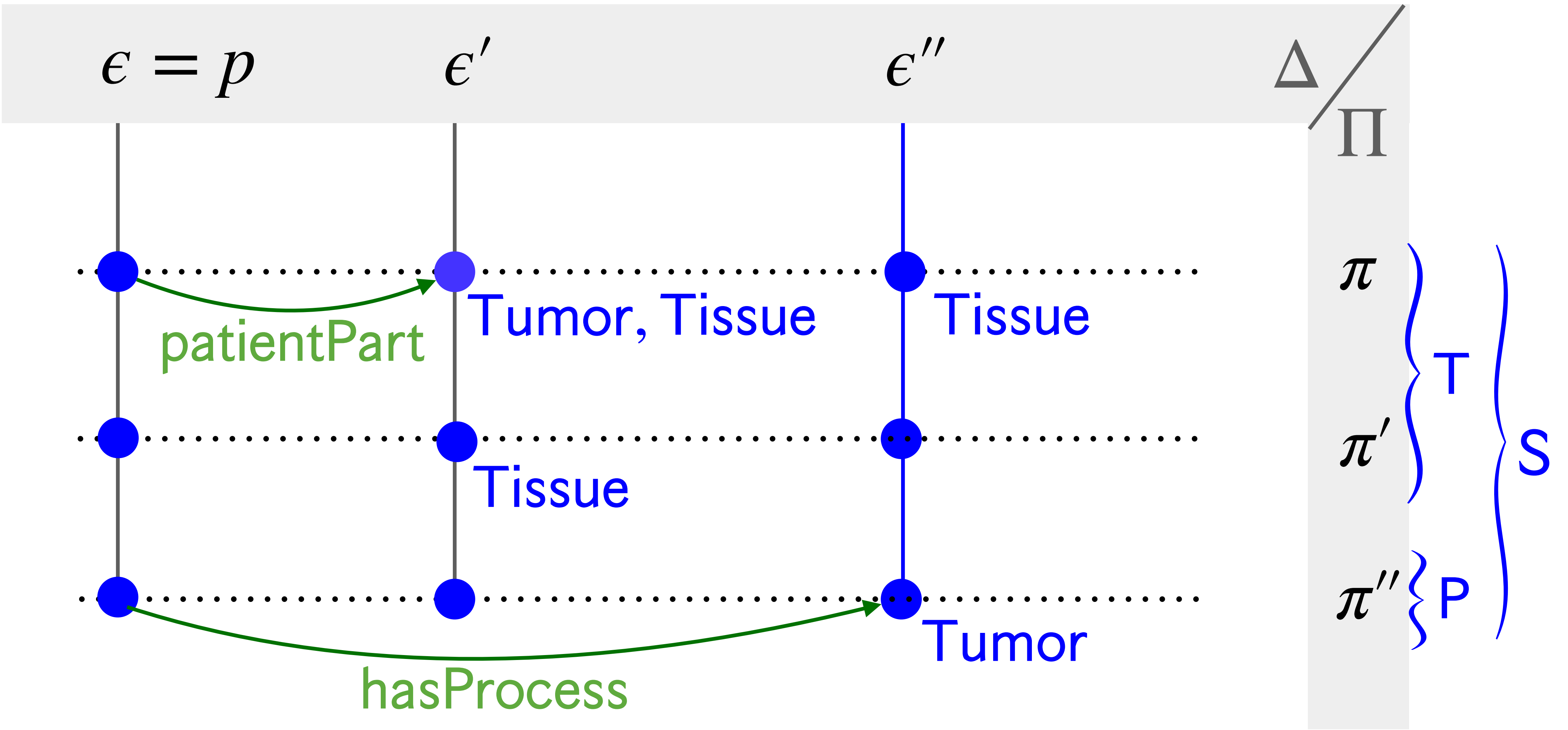

Complexity and Automated Reasoning

Tractable Reasoning in $\mathbb{S}_{\mathscr{EL}+}$

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- Nominal Concepts → ExpTime-hard

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Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

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(1) Normalisation:

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  - $s' \preceq s$ $\quad$ $s_1 \cap s_2 \preceq s$
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  - $\Box_s (C \sqsubseteq \Box_u D)$ $\quad$ $\Box_s (C \sqsubseteq \Diamond_u D)$
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* can be rewritten (with a fresh concept D) to
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* can be rewritten (with a fresh concept D) to

$\square_{\mathsf{t}}[A \sqsubseteq \square_{\mathsf{s}} D]$ and $\square_{\mathsf{s}}[D \sqcap B \sqsubseteq C]]$

Then replace:

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\quad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\square_{\mathsf{s}}(C \sqsubseteq D)$ $\quad$ $\square_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
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  - $\square_{\mathsf{s}}(C \sqsubseteq \square_{\mathsf{u}} D)$ $\quad$ $\square_{\mathsf{s}}(C \sqsubseteq \lozenge_{\mathsf{u}} D)$
- RIAs:
  - $\square_{\mathsf{s}}(R' \sqsubseteq R)$ $\quad$ $\square_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
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- $\square_\mathsf{s} C(a)$ by $\square_\mathsf{s}[\{a\} \sqsubseteq \square_\mathsf{s}[\top \Rightarrow C]]$

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

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  - $\Box_s (C \sqsubseteq \Box_u D)$ $\quad$ $\Box_s (C \sqsubseteq \Diamond_u D)$
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Then replace:

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- $\Box_s C(a)$ by $\Box_s [\{a\} \sqsubseteq \Box_s [\top \Rightarrow C]]$

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

Tautologies

$(T.1)\ \dfrac{}{\mathsf{s} \preceq *}$ $(T.2)\ \dfrac{}{\mathsf{s} \preceq \mathsf{s}}$ $(T.3)\ \dfrac{}{\Box_*[\top \sqsubseteq \Box_*[C \Rightarrow C]]}$ $(T.4)\ \dfrac{}{\Box_*[\top \sqsubseteq \Box_*[C \Rightarrow \top]]}$ $(T.5)\ \dfrac{}{\Box_*[R \sqsubseteq R]}$

Standpoint hierarchy rules (for all s $\in N_S$, ξ being any extended GCI, RIA, or role assertion)

$(S.1)\ \dfrac{\mathsf{s} \preceq \mathsf{s}' \quad \mathsf{s}' \preceq \mathsf{s}''}{\mathsf{s} \preceq \mathsf{s}''}$ $(S.2)\ \dfrac{\mathsf{s} \preceq \mathsf{s}_1 \quad \mathsf{s} \preceq \mathsf{s}_2 \quad \mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}'}{\mathsf{s} \preceq \mathsf{s}'}$ $(S.3)\ \dfrac{\Box_{\mathsf{s}'}\xi \quad \mathsf{s} \preceq \mathsf{s}'}{\Box_{\mathsf{s}}\xi}$ $(S.4)\ \dfrac{\Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}'}[D \Rightarrow E]] \quad \mathsf{s} \preceq \mathsf{s}'}{\Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}$

Internal inferences for extended GCIs

$(I.1)\ \dfrac{\Box_{\mathsf{s}}[C \sqsubseteq \Box_{\mathsf{s}}[\top \Rightarrow D]]}{\Box_*[\top \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]]}$ $(I.2)\ \dfrac{\Box_{\mathsf{u}}[\top \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]]}{\Box_*[\top \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]]}$

Role subsumptions

$(R.1)\ \dfrac{\Box_{\mathsf{s}}[R \sqsubseteq R''] \quad \Box_{\mathsf{s}}[R'' \sqsubseteq R']}{\Box_{\mathsf{s}}[R \sqsubseteq R']}$

Forward chaining

$(C.1)\ \dfrac{\Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow D]] \quad \Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}{\Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[C \Rightarrow E]]}$ $(C.2)\ \dfrac{\Box_{\mathsf{u}}[\top \sqsubseteq \Box_{\mathsf{t}}[B \Rightarrow C]] \quad \Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}{\Box_{\mathsf{t}}[B \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}$

$(C.3)\ \dfrac{\Box_{\mathsf{u}}[\top \sqsubseteq \Box_{\mathsf{t}}[C \Rightarrow D]] \quad \Box_{\mathsf{t}}[D \sqsubseteq \Diamond_{\mathsf{s}}E]}{\Box_{\mathsf{t}}[C \sqsubseteq \Diamond_{\mathsf{s}}E]}$ $(C.4)\ \dfrac{\Box_{\mathsf{t}}[C \sqsubseteq \Diamond_{\mathsf{s}}D] \quad \Box_{\mathsf{t}}[C \sqsubseteq \Box_{\mathsf{s}}[D \Rightarrow E]]}{\Box_{\mathsf{t}}[C \sqsubseteq \Diamond_{\mathsf{s}}E]}$

... (26 more rules)

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$

Tautologies

$$(T.1)\ \frac{}{\mathsf{s} \preceq *} \qquad (T.2)\ \frac{}{\mathsf{s} \preceq \mathsf{s}} \qquad (T.3)\ \frac{}{\Box_*[\top \sqsubseteq \Box_*[C \Rightarrow C]]} \qquad (T.4)\ \frac{}{\Box_*[\top \sqsubseteq \Box_*[C \Rightarrow \top]]} \qquad (T.5)\ \frac{}{\Box_*[R \sqsubseteq R]}$$

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Role subsumptions

$$(R.1)\ \frac{\Box_{\mathsf{s}}[R \sqsubseteq R''] \quad \Box_{\mathsf{s}}[R'' \sqsubseteq R']}{\Box_{\mathsf{s}}[R \sqsubseteq R']}$$

Forward chaining

If $\Box_*[\top \sqsubseteq \Box_*[\top \Rightarrow \bot]] \notin \mathscr{K}^{\vdash}$, then $\mathscr{K}$ is satisfiable

Decision Calculus for $\mathbb{S}_{\mathscr{EL}+}$ (Proofs)

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Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathscr{EL}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

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- We prove the existence of a model whenever $\Box_* [\top \sqsubseteq \Box_* [\top \Rightarrow \bot]] \notin \mathscr{K}^{\vdash}$.
- This model is canonical in a sense but it will typically be infinite.

Conclusions and Future Work

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➡ Managing perspectives is interesting in knowledge integration scenarios

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- ➡ Standpoint $\mathscr{EL}+$ is tractable
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Future Work:

- ➡ Calculus optimisation and efficient implementations
- ➡ Reasoning with more expressive languages (eg. $\mathscr{SHIQ}$)
- ➡ Towards conceptual modelling with standpoints for knowledge integration challenges

The end.

Labels example

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$\Box_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L [\text{Tumor}] \sqsubseteq \Box_L [\text{Tissue}]$

$\Diamond_H [\text{Tumor}] \sqsubseteq \Box_H [\text{Process}]$

$\Diamond_S \neg\text{Tumor}(a)$

$(L \cup H) \preceq S$

(It could be a Tumor according to someone else)

$\Box_L \text{ Tumor}(a)$

Labels example

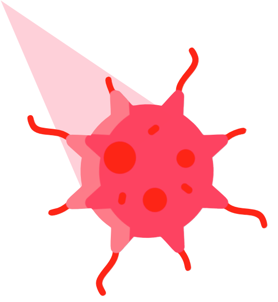
$\square_S\,[\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot]$

$\lozenge_L\,[\mathsf{Tumor}] \sqsubseteq \square_L\,[\mathsf{Tissue}]$

$\lozenge_H\,[\mathsf{Tumor}] \sqsubseteq \square_H\,[\mathsf{Process}]$

$\lozenge_S\,\neg\mathsf{Tumor}(a)$

$(L \cup H) \preceq S$



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$\square_L\,\mathsf{Tumor}(a)$

$[\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot]$

$\mathsf{Tumor}_L \sqsubseteq \mathsf{Tissue}$, $\mathsf{Tumor}_L \sqsubseteq \mathsf{Tumor}$

$\mathsf{Tumor}_H \sqsubseteq \mathsf{Process}$, $\mathsf{Tumor}_H \sqsubseteq \mathsf{Tumor}$

$\neg\mathsf{Tumor}(a)$

Infer: (It cannot be a Tumor according to anyone)

$\neg\mathsf{Tumor}_L(a)$

$\neg\mathsf{Tumor}_H(a)$