

Formal Concept Analysis

Exercise Sheet 7, Winter Semester 2016/17

Definition (closure system and closure operator).

(a) A set $\mathfrak{A} \subseteq \mathfrak{P}(M)$ is a closure system on the set M , iff $M \in \mathfrak{A}$ and $\mathfrak{X} \subseteq \mathfrak{A} \implies \bigcap \mathfrak{X} \in \mathfrak{A}$.

(b) A closure operator φ on M is a map φ which maps each subset $X \subseteq M$ onto the corresponding closure $\varphi(X) \subseteq M$ such that

1) $X \subseteq \varphi(X)$ (extensive)

2) $X \subseteq Y \implies \varphi(X) \subseteq \varphi(Y)$ (monotone)

3) $\varphi(\varphi(X)) = \varphi(X)$ (idempotent)

holds.

Exercise 1

Prove the following statements:

- a) For any closure system \mathfrak{A} on some set M , the mapping $\varphi_{\mathfrak{A}} : X \mapsto \bigcap_{X \subseteq Y \in \mathfrak{A}} Y$ is a closure operator on M .
- b) For any closure operator φ on some set M , the family $\mathfrak{A}_{\varphi} := \{\varphi(X) \mid X \subseteq M\}$ is a closure system on M .

Exercise 2 (closure system)

Regard the “family context” $\mathbb{K} := (\{\text{father, mother, daughter, son}\}, \{\text{old, young, male, female}\}, \{(\text{father, old}), (\text{father, male}), (\text{mother, old}), (\text{mother, female}), (\text{daughter, young}), (\text{daughter, female}), (\text{son, young}), (\text{son, male})\})$.

- a) Explicitly list the elements of the map $\varphi : \mathfrak{P}(M) \rightarrow \mathfrak{P}(M)$ with $\varphi : B \mapsto B''$ and verify that φ is a closure operator.
- b) Verify that the set of all concept intents of the family context is a closure system.
- c) Draw a line diagram of the powerset of $\{\text{father, mother, daughter, son}\}$ and highlight the sets that have the same closure. Compare the diagram with the diagram of the concept lattice of the family context.

Exercise 3 (Next-Closure)

| | old (1) | young (2) | male (3) | female (4) |
|----------|---------|-----------|----------|------------|
| father | × | | × | |
| mother | × | | | × |
| son | | × | × | |
| daughter | | × | | × |

Compute all concept intents of the above “family context” using the Next-Closure algorithm. Compare your result with the concept intents from Exercise 2.

| A | i | $(A \cap \{1, 2, \dots, i-1\}) \cup \{i\}$ $A + i$ | $(A+i)''$ $A \oplus i$ | $A <_i A \oplus i?$ | new intent |
|---|-----|---|---------------------------|---------------------|------------|
| | | | | | |