



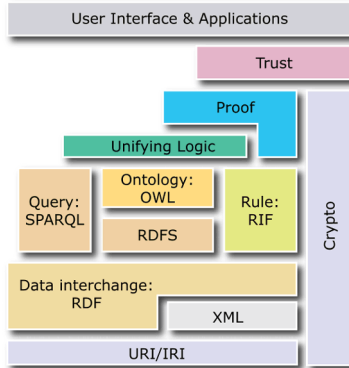
TECHNISCHE
UNIVERSITÄT
DRESDEN

FOUNDATIONS OF SEMANTIC WEB TECHNOLOGIES

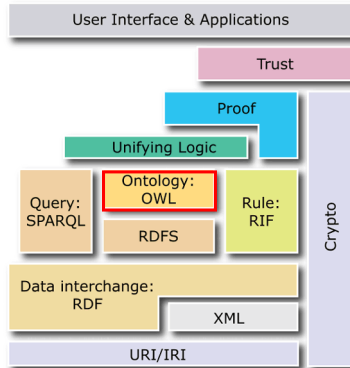
Hypertableau I

Sebastian Rudolph

Hypertableau I



Hypertableau I



Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\}$$

$$L(b_2) = \{C_{\mathcal{T}}\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\}$$

$$L(b_2) = \{C_{\mathcal{T}}\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \dots \dots \dots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\} \cup \{\neg A^{n+2}\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\} \cup \{\neg A^{n+2}\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\} \cup \{\forall r.(\neg A)^{n+3}\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \cdots \cdots \cdots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\} \cup \{\neg A^{n+2}\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\} \cup \{\neg A^{2n}\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\} \cup \{\forall r.(\neg A)^{n+3}\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \dots \dots \dots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\} \cup \{\neg A^{n+2}\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\} \cup \{\neg A^{2n}\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\} \cup \{\forall r.(\neg A)^{n+3}\}$$

$$\vdots \quad \quad \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\} \cup \{\forall r.(\neg A)^{2n+1}\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \dots \dots \dots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\} \cup \{\neg A^{n+2}\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\} \cup \{\neg A^{2n}\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\} \cup \{\neg A^{2n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\} \cup \{\forall r.(\neg A)^{n+3}\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\} \cup \{\forall r.(\neg A)^{2n+1}\} \cup \{A^{2n+1}\}$$

Example Standard Tableau

$$C_{\mathcal{T}} = \forall r.(\neg A) \sqcup A$$

$$a_0 \xrightarrow{r} b_1 \xrightarrow{r} a_1 \xrightarrow{r} b_2 \dots \dots \dots \rightarrow a_{n-1} \xrightarrow{r} b_n \xrightarrow{r} a_n$$

$$L(a_0) = \{\neg A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^1\}$$

$$L(a_1) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^2\} \cup \{\neg A^{n+2}\}$$

$$\vdots \quad \vdots$$

$$L(a_{n-1}) = \{C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^n\} \cup \{\neg A^{2n}\}$$

$$L(a_n) = \{A, C_{\mathcal{T}}\} \cup \{\forall r.(\neg A)^{n+1}\} \cup \{\neg A^{2n+1}\}$$

$$L(b_1) = \{C_{\mathcal{T}}\} \cup \{\neg A^1\} \cup \{\forall r.(\neg A)^{n+2}\}$$

$$L(b_2) = \{C_{\mathcal{T}}\} \cup \{\neg A^2\} \cup \{\forall r.(\neg A)^{n+3}\}$$

$$\vdots \quad \vdots$$

$$L(b_n) = \{C_{\mathcal{T}}\} \cup \{\neg A^n\} \cup \{\forall r.(\neg A)^{2n+1}\} \cup \{A^{2n+1}\}$$

Is that Necessarily So?

- the algorithm constructs exponentially many branches, despite dependency directed backtracking
- translation of the formula into FOL:

$$\begin{aligned} & \forall r. (\neg A) \sqcup A \\ &= \forall x, y. [r(x, y) \wedge A(y) \rightarrow A(x)] \\ &= \forall x, y. [\neg r(x, y) \vee \neg A(y) \vee A(x)] \end{aligned}$$

- note: the formula does not have real non-determinism (Horn-clause)
- hypertableau exploits this

Idea Hypertableau

- translate KB axioms into FOL
 - rewrite axioms to obtain formulae of a certain structure
- axioms are translated such that non-determinism is avoided, if possible
- the formulae thus obtained become rules for constructing a model abstraction

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A \sqsubseteq \exists r.B$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A(x) \rightarrow A'(x)$$

$$A \sqsubseteq \exists r.B$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A(x) \rightarrow A'(x)$$

$$A \sqsubseteq \exists r.B$$

$$A(x) \rightarrow \exists r.B(x)$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A(x) \rightarrow A'(x)$$

$$A \sqsubseteq \exists r.B$$

$$A(x) \rightarrow \exists r.B(x)$$

$$D \sqsubseteq E \sqcup F$$

$$D(x) \rightarrow E(x) \vee F(x)$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A \sqsubseteq \exists r.B$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

$$A(x) \rightarrow A'(x)$$

$$A(x) \rightarrow \exists r.B(x)$$

$$D(x) \rightarrow E(x) \vee F(x)$$

$$F(x) \rightarrow \perp(x)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A \sqsubseteq \exists r.B$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

$$A(x) \rightarrow A'(x)$$

$$A(x) \rightarrow \exists r.B(x)$$

$$D(x) \rightarrow E(x) \vee F(x)$$

$$F(x) \rightarrow \perp(x)$$

$$r(x, y) \rightarrow C(x)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1 Translation into Clauses

$$A \sqsubseteq A'$$

$$A \sqsubseteq \exists r.B$$

$$D \sqsubseteq E \sqcup F$$

$$F \sqsubseteq \perp$$

$$\exists r.T \sqsubseteq C$$

$$A(a)$$

$$(D \sqcap \neg B)(d)$$

$$A(x) \rightarrow A'(x)$$

$$A(x) \rightarrow \exists r.B(x)$$

$$D(x) \rightarrow E(x) \vee F(x)$$

$$F(x) \rightarrow \perp(x)$$

$$r(x, y) \rightarrow C(x)$$

$$\rightarrow A(a)$$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

1 Translation into Clauses

$A \sqsubseteq A'$	$A(x) \rightarrow A'(x)$
$A \sqsubseteq \exists r.B$	$A(x) \rightarrow \exists r.B(x)$
$D \sqsubseteq E \sqcup F$	$D(x) \rightarrow E(x) \vee F(x)$
$F \sqsubseteq \perp$	$F(x) \rightarrow \perp(x)$
$\exists r.\top \sqsubseteq C$	$r(x, y) \rightarrow C(x)$
$A(a)$	$\rightarrow A(a)$
$(D \sqcap \neg B)(d)$	$\rightarrow D(d)$
	$\rightarrow \neg B(d)$

- existential quantifiers treated as in the tableau

Simple Hypertableau Example

a

d

$$L(a) = \{A\}$$

$$L(d) = \{D, \neg B\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

a

d

$$L(a) = \{A\}$$

$$L(d) = \{D, \neg B\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

 a d

$$L(a) = \{A\} \cup \{A'\}$$

$$L(d) = \{D, \neg B\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

 a d

$$L(a) = \{A\} \cup \{A'\}$$

$$L(d) = \{D, \neg B\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

a

d

$$L(a) = \{A\} \cup \{A'\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

a

d

$$L(a) = \{A\} \cup \{A'\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

a

d

$$L(a) = \{A\} \cup \{A'\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example

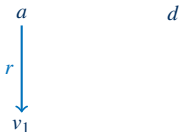
a *d*

$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example



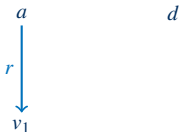
$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$L(v_1) = \{B\}$$

$$\begin{aligned}
 A(x) &\rightarrow A'(x) \\
 A(x) &\rightarrow \exists r.B(x) \\
 D(x) &\rightarrow E(x) \vee F(x) \\
 F(x) &\rightarrow \perp(x) \\
 r(x, y) &\rightarrow C(x) \\
 &\rightarrow A(a) \\
 &\rightarrow D(d) \\
 &\rightarrow \neg B(d)
 \end{aligned}$$

Simple Hypertableau Example



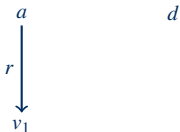
$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$L(v_1) = \{B\}$$

$$\begin{aligned} A(x) &\rightarrow A'(x) \\ A(x) &\rightarrow \exists r.B(x) \\ D(x) &\rightarrow E(x) \vee F(x) \\ F(x) &\rightarrow \perp(x) \\ r(x, y) &\rightarrow C(x) \\ &\rightarrow A(a) \\ &\rightarrow D(d) \\ &\rightarrow \neg B(d) \end{aligned}$$

Simple Hypertableau Example



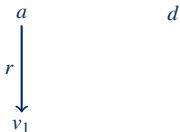
$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\} \cup \{C\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$L(v_1) = \{B\}$$

$$\begin{aligned}
 A(x) &\rightarrow A'(x) \\
 A(x) &\rightarrow \exists r.B(x) \\
 D(x) &\rightarrow E(x) \vee F(x) \\
 F(x) &\rightarrow \perp(x) \\
 r(x, y) &\rightarrow C(x) \\
 &\rightarrow A(a) \\
 &\rightarrow D(d) \\
 &\rightarrow \neg B(d)
 \end{aligned}$$

Simple Hypertableau Example



$$L(a) = \{A\} \cup \{A'\} \cup \{\exists r.B\} \cup \{C\}$$

$$L(d) = \{D, \neg B\} \cup \{E^1\}$$

$$L(v_1) = \{B\}$$

$$\begin{array}{l}
 A(x) \rightarrow A'(x) \\
 A(x) \rightarrow \exists r.B(x) \\
 D(x) \rightarrow E(x) \vee F(x) \\
 F(x) \rightarrow \perp(x) \\
 r(x, y) \rightarrow C(x) \\
 \rightarrow A(a) \\
 \rightarrow D(d) \\
 \rightarrow \neg B(d)
 \end{array}$$

- no more rules applicable \rightsquigarrow satisfiability has been shown
- the only thing left from the tableau rules: an analogue of the \exists -rule

Agenda

- Motivation
- **Recap: Translation into FOL**
- Structural Transformation
- Translation into Clauses
- Summary

Translation into FOL

translation of TBox axioms into FOL via the mapping π with C, D complex classes, r a role and A an atomic class:

$$\pi(C \sqsubseteq D) = \forall x. (\pi_x(C) \rightarrow \pi_x(D)) \quad \pi(C \equiv D) = \forall x. (\pi_x(C) \leftrightarrow \pi_x(D))$$

$$\pi_x(A) = A(x)$$

$$\pi_y(A) = A(y)$$

$$\pi_x(\neg C) = \neg \pi_x(C)$$

$$\pi_y(\neg C) = \neg \pi_y(C)$$

$$\pi_x(C \sqcap D) = \pi_x(C) \wedge \pi_x(D)$$

$$\pi_y(C \sqcap D) = \pi_y(C) \wedge \pi_y(D)$$

$$\pi_x(C \sqcup D) = \pi_x(C) \vee \pi_x(D)$$

$$\pi_y(C \sqcup D) = \pi_y(C) \vee \pi_y(D)$$

$$\pi_x(\forall r. C) = \forall y. (r(x, y) \rightarrow \pi_y(C))$$

$$\pi_y(\forall r. C) = \forall x. (r(y, x) \rightarrow \pi_x(C))$$

$$\pi_x(\exists r. C) = \exists y. (r(x, y) \wedge \pi_y(C))$$

$$\pi_y(\exists r. C) = \exists x. (r(y, x) \wedge \pi_x(C))$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)))$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)))$$
$$\forall x. [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))]$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\begin{aligned} & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\ \forall x. & [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \end{aligned}$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\begin{aligned} & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\ & \forall x. [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\ & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \\ & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D \sqcup \exists r. D)))] \end{aligned}$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\begin{aligned} & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\ \forall x. & [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D \sqcup \exists r. D)))] \\ \forall x. & [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D) \vee \pi_x(\exists r. D)))] \end{aligned}$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\begin{aligned} & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\ & \forall x. [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\ & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \\ & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D \sqcup \exists r. D)))] \\ & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D) \vee \pi_x(\exists r. D)))] \\ & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow D(x) \vee \exists y. (r(x, y) \wedge \pi_y(D))))] \end{aligned}$$

Motivation Normal Form

- the given translation creates rather complex formulae for complex axioms

$$\begin{aligned}
 & \pi(C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))) \\
 & \forall x. [\pi_x(C) \rightarrow \pi_x(\exists r. (\forall s. (D \sqcup \exists r. D)))] \\
 & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \pi_y(\forall s. (D \sqcup \exists r. D)))] \\
 & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D \sqcup \exists r. D)))] \\
 & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow \pi_x(D) \vee \pi_x(\exists r. D)))] \\
 & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow D(x) \vee \exists y. (r(x, y) \wedge \pi_y(D))))] \\
 & \forall x. [C(x) \rightarrow \exists y. (r(x, y) \wedge \forall x. (s(y, x) \rightarrow D(x) \vee \exists y. (r(x, y) \wedge D(y))))]
 \end{aligned}$$

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$C \sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D))$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$\begin{aligned} C &\sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)) \\ \rightsquigarrow C &\sqsubseteq \exists r. Q_1 \\ Q_1 &\equiv \forall s. (D \sqcup \exists r. D) \end{aligned}$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$\begin{aligned} C &\sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)) \\ \rightsquigarrow C &\sqsubseteq \exists r. Q_1 \\ Q_1 &\equiv \forall s. (D \sqcup \exists r. D) \\ \rightsquigarrow Q_1 &\equiv \forall s. Q_2 \\ Q_2 &\equiv D \sqcup \exists r. D \end{aligned}$$

Structural Transformation

- Structural Transformation introduces new concepts for complex subconcepts

$$\begin{aligned} C &\sqsubseteq \exists r. (\forall s. (D \sqcup \exists r. D)) \\ \rightsquigarrow C &\sqsubseteq \exists r. Q_1 \\ Q_1 &\equiv \forall s. (D \sqcup \exists r. D) \\ \rightsquigarrow Q_1 &\equiv \forall s. Q_2 \\ Q_2 &\equiv D \sqcup \exists r. D \\ \rightsquigarrow Q_2 &\equiv D \sqcup Q_3 \\ \rightsquigarrow Q_3 &\equiv \exists r. D \end{aligned}$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r. (\forall r. B)$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r. (\forall r. B)$$

$$\rightsquigarrow A \sqsubseteq \forall r. Q$$

$$\rightsquigarrow Q \sqsubseteq \forall r. B$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r. (\forall r. B)$$

$$\rightsquigarrow A \sqsubseteq \forall r. Q$$

$$\rightsquigarrow Q \sqsubseteq \forall r. B \quad \checkmark$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r. (\forall r. B)$$

$$\exists r. (A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow A \sqsubseteq \forall r. Q$$

$$\rightsquigarrow Q \sqsubseteq \forall r. B \quad \checkmark$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r. (\forall r. B)$$

$$\rightsquigarrow A \sqsubseteq \forall r. Q$$

$$\rightsquigarrow Q \sqsubseteq \forall r. B \quad \checkmark$$

$$\exists r. (A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \exists r. Q \sqsubseteq C$$

$$Q \sqsubseteq A \sqcap B$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$A \sqsubseteq \forall r. (\forall r. B)$$

$$\rightsquigarrow A \sqsubseteq \forall r. Q$$

$$\rightsquigarrow Q \sqsubseteq \forall r. B \quad \checkmark$$

$$\exists r. (A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \exists r. Q \sqsubseteq C$$

$$Q \sqsubseteq A \sqcap B$$

$$\rightsquigarrow Q \sqsubseteq A$$

$$\rightsquigarrow Q \sqsubseteq B$$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$\begin{array}{ll}
 A \sqsubseteq \forall r. (\forall r. B) & \exists r. (A \sqcap B) \sqsubseteq C \\
 \rightsquigarrow A \sqsubseteq \forall r. Q & \rightsquigarrow \exists r. Q \sqsubseteq C \\
 \rightsquigarrow Q \sqsubseteq \forall r. B \quad \checkmark & Q \sqsubseteq A \sqcap B \\
 & \rightsquigarrow Q \sqsubseteq A \\
 & \rightsquigarrow Q \sqsubseteq B
 \end{array}$$

Well. Is that correct? Let $\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$\begin{array}{ll}
 A \sqsubseteq \forall r. (\forall r. B) & \exists r. (A \sqcap B) \sqsubseteq C \\
 \rightsquigarrow A \sqsubseteq \forall r. Q & \rightsquigarrow \exists r. Q \sqsubseteq C \\
 \rightsquigarrow Q \sqsubseteq \forall r. B \quad \checkmark & Q \sqsubseteq A \sqcap B \\
 & \rightsquigarrow Q \sqsubseteq A \\
 & \rightsquigarrow Q \sqsubseteq B
 \end{array}$$

Well. Is that correct? Let $\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$

- In the context of a TBox with the original axiom, the ABox was contradictory.

Optimization of Structural Transformation

- do we have to introduce equivalence axioms?

$$\begin{array}{ll}
 A \sqsubseteq \forall r.(\forall r.B) & \exists r.(A \sqcap B) \sqsubseteq C \\
 \rightsquigarrow A \sqsubseteq \forall r.Q & \rightsquigarrow \exists r.Q \sqsubseteq C \\
 \rightsquigarrow Q \sqsubseteq \forall r.B \quad \checkmark & Q \sqsubseteq A \sqcap B \\
 & \rightsquigarrow Q \sqsubseteq A \\
 & \rightsquigarrow Q \sqsubseteq B
 \end{array}$$

Well. Is that correct? Let $\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$

- In the context of a TBox with the original axiom, the ABox was contradictory.
- In the context of a TBox with the rewritten axiom, the ABox is satisfiable

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use \sqsupseteq

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use \sqsupseteq

$$\exists r.(A \sqcap B) \sqsubseteq C$$

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use \sqsupseteq

$$\exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \exists r.Q \sqsubseteq C$$

$$Q \sqsupseteq A \sqcap B$$

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use \sqsupseteq

$$\exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \exists r.Q \sqsubseteq C$$

$$Q \sqsupseteq A \sqcap B$$

$$\rightsquigarrow A \sqcap B \sqsubseteq Q$$

Polarity for Optimized Transformation

- we have to take care if subexpressions occur “positively” or “negatively”
- $A \sqsubseteq B$ is just $\neg A \sqcup B$
- thus, A occurs negatively in the axiom and B positively
- when replacing a negatively occurring concept, we have to use \sqsupseteq

$$\exists r.(A \sqcap B) \sqsubseteq C$$

$$\rightsquigarrow \exists r.Q \sqsubseteq C$$

$$Q \sqsupseteq A \sqcap B$$

$$\rightsquigarrow A \sqcap B \sqsubseteq Q$$

$$C_{\mathcal{T}} = (\forall r.(\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\}$$

$$L(b) = \{A, B\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\} \\ \cup \{\neg A\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

~~$$\cup \{\neg A\}$$~~

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

$$\cup \{\neg A\} \cup \{\neg B\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

$$\cup \{\neg A\} \cup \{\neg B\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

$$\cup \{\neg A\} \cup \{\neg B\} \cup \{Q\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

$$\cup \{\neg A\} \cup \{\neg B\} \cup \{Q\}$$

Tableau for Unsatisfiability

$$C_{\mathcal{T}} = (\forall r. (\neg Q) \sqcup C) \sqcap (\neg A \sqcup \neg B \sqcup Q)$$

$$\mathcal{A} = \{r(a, b), A(b), B(b), \neg C(a)\}$$



$$L(a) = \{\neg C\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q), \neg A\}$$

$$L(b) = \{A, B\} \cup \{\neg Q\} \cup \{C_{\mathcal{T}}, \forall r. (\neg Q) \sqcup C, \neg A \sqcup \neg B \sqcup Q, \forall r. (\neg Q)\}$$

~~$$\cup \{\neg A\} \cup \{\neg B\} \cup \{Q\}$$~~

no further choice options
 \rightsquigarrow the KB is unsatisfiable

Optimized Structural Transformation

- we now want to define structural transformation formally
- we want to introduce just one new concept name per subexpression
- goal: rewrite TBox into an equisatisfiable TBox containing only “simple” axioms

Polarity of Concepts

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E ,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D ,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D .

Polarity of Concepts

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E ,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D ,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D .

A concept occurs positively (negatively) in an (\mathcal{ALC}) TBox \mathcal{T} , if C occurs positively (negatively) in an axiom in \mathcal{T} .

Polarity of Concepts

We define the polarity of a concept C inside a formula as follows:

- C occurs in C positively,
- C occurs in $\neg D$ positively (negatively) if C occurs in D negatively (positively),
- C occurs in $D \sqcap E$ or $D \sqcup E$ positively (negatively), if C occurs positively (negatively) in D or E ,
- C occurs in $\exists r.D$ or $\forall r.D$ positively (negatively), if C occurs positively (negatively) in D ,
- C occurs in $D \sqsubseteq E$ positively (negatively), if C occurs positively (negatively) in E or negatively (positively) in D .

A concept occurs positively (negatively) in an (\mathcal{ALC}) TBox \mathcal{T} , if C occurs positively (negatively) in an axiom in \mathcal{T} .

\rightsquigarrow a concept may occur both positively and negatively in an axiom

Optimized Transformation with Polarity

Let \mathcal{T} be an \mathcal{ALC} TBox. For every concept (sub-)expression C in \mathcal{T} , we introduce a fresh atomic concept A_C and define the function $\text{st}(C)$ as follows:

$$\begin{array}{lll} \text{st}(A) = A & \text{st}(\neg C) = \neg A_C & \text{st}(\exists r.C) = \exists r.A_C \\ \text{st}(\top) = \top & \text{st}(C \sqcap D) = A_C \sqcap A_D & \text{st}(\forall r.C) = \forall r.A_C \\ \text{st}(\perp) = \perp & \text{st}(C \sqcup D) = A_C \sqcup A_D & \end{array}$$

The result of the structural transformation of a TBox \mathcal{T} is a TBox \mathcal{T}' with the following axioms:

- $A_C \sqsubseteq \text{st}(C)$ for every concept C occurring positively in \mathcal{T} ,
- $\text{st}(C) \sqsubseteq A_C$ for every concept C occurring negatively in \mathcal{T} ,
- $A_C \sqsubseteq A_D$ for every GCI $C \sqsubseteq D \in \mathcal{T}$.

Simplification of Axioms

we can now use known equivalences to simplify the axioms further:

$$\{C \sqsubseteq D \sqcap E\} \equiv \{C \sqsubseteq D, C \sqsubseteq E\}$$

$$\{C \sqcup D \sqsubseteq E\} \equiv \{C \sqsubseteq E, D \sqsubseteq E\}$$

Result of the Structural Transformation

by virtue of structural transformation and the known equivalences, we can rewrite an \mathcal{ALC} KB into an equisatisfiable one, which contains only axioms of the following shape (A and B being atomic):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$

$$A \sqsubseteq \exists r.B$$

$$A \sqsubseteq \forall r.B$$

$$\exists r.A \sqsubseteq B$$

$$\forall r.A \sqsubseteq B$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$\underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2} \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_9} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$A \sqcap D \sqsubseteq C_1$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2} \sqcup \forall r.A}_{C_9}$$

$$\begin{aligned}
 A \sqcap D &\sqsubseteq C_1 \\
 C_2 &\sqsubseteq C \sqcap E
 \end{aligned}$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A \sqcap \underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_7}} \sqsubseteq \underbrace{B \sqcup \underbrace{(\exists r.(C \sqcap E) \sqcap F)}_{C_5} \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

$$\begin{aligned}
 A \sqcap D &\sqsubseteq C_1 \\
 C_2 &\sqsubseteq C \sqcap E \\
 \forall r.C_1 &\sqsubseteq C_3
 \end{aligned}$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$\underbrace{\hspace{15em}}_{C_9}$

$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C \sqcap E$$

$$\forall r.C_1 \sqsubseteq C_3$$

$$\exists s.C_1 \sqsubseteq C_4$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$\underbrace{\hspace{15em}}_{C_9}$

$$\begin{aligned}
 A \sqcap D &\sqsubseteq C_1 \\
 C_2 &\sqsubseteq C \sqcap E \\
 \forall r.C_1 &\sqsubseteq C_3 \\
 \exists s.C_1 &\sqsubseteq C_4 \\
 C_5 &\sqsubseteq \exists r.C_3
 \end{aligned}$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2}}_{C_9} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\begin{aligned}
 A \sqcap D &\sqsubseteq C_1 \\
 C_2 &\sqsubseteq C \sqcap E \\
 \forall r.C_1 &\sqsubseteq C_3 \\
 \exists s.C_1 &\sqsubseteq C_4 \\
 C_5 &\sqsubseteq \exists r.C_3
 \end{aligned}$$

$$C_6 \sqsubseteq \forall r.A$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_9} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C \sqcap E$$

$$\forall r.C_1 \sqsubseteq C_3$$

$$\exists s.C_1 \sqsubseteq C_4$$

$$C_5 \sqsubseteq \exists r.C_3$$

$$C_6 \sqsubseteq \forall r.A$$

$$C_3 \sqcap A \sqcap C_4 \sqsubseteq C_7$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_9} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C \sqcap E$$

$$\forall r.C_1 \sqsubseteq C_3$$

$$\exists s.C_1 \sqsubseteq C_4$$

$$C_5 \sqsubseteq \exists r.C_3$$

$$C_6 \sqsubseteq \forall r.A$$

$$C_3 \sqcap A \sqcap C_4 \sqsubseteq C_7$$

$$C_8 \sqsubseteq C_5 \sqcap F$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\underbrace{B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C \sqcap E$$

$$\forall r.C_1 \sqsubseteq C_3$$

$$\exists s.C_1 \sqsubseteq C_4$$

$$C_5 \sqsubseteq \exists r.C_3$$

$$C_6 \sqsubseteq \forall r.A$$

$$C_3 \sqcap A \sqcap C_4 \sqsubseteq C_7$$

$$C_8 \sqsubseteq C_5 \sqcap F$$

$$C_9 \sqsubseteq B \sqcup C_8 \sqcup C_6$$

Example: Optimized Structural Transformation

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\underbrace{B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E) \sqcap F}_{C_5}}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}}_{C_9}$$

$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C \sqcap E$$

$$\forall r.C_1 \sqsubseteq C_3$$

$$\exists s.C_1 \sqsubseteq C_4$$

$$C_5 \sqsubseteq \exists r.C_3$$

$$C_6 \sqsubseteq \forall r.A$$

$$C_3 \sqcap A \sqcap C_4 \sqsubseteq C_7$$

$$C_8 \sqsubseteq C_5 \sqcap F$$

$$C_9 \sqsubseteq B \sqcup C_8 \sqcup C_6$$

$$C_7 \sqsubseteq C_9$$

Example: Optimized Structural Transformation

we still can apply the simplification rules:

$$\overbrace{\underbrace{\underbrace{\forall r.(A \sqcap D)}_{C_3} \sqcap A}_{C_1} \sqcap \underbrace{\underbrace{\exists s.(A \sqcap D)}_{C_4}}_{C_1}}^{C_7} \sqsubseteq B \sqcup \underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2}}_{C_8} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$\underbrace{\underbrace{\underbrace{\exists r.(C \sqcap E)}_{C_5} \sqcap F}_{C_2}}_{C_9} \sqcup \underbrace{\forall r.A}_{C_6}$$

$$A \sqcap D \sqsubseteq C_1$$

$$C_2 \sqsubseteq C$$

$$\forall r.C_1 \sqsubseteq C_3$$

$$\exists s.C_1 \sqsubseteq C_4$$

$$C_5 \sqsubseteq \exists r.C_3$$

$$C_2 \sqsubseteq E$$

$$C_6 \sqsubseteq \forall r.A$$

$$C_3 \sqcap A \sqcap C_4 \sqsubseteq C_7$$

$$C_8 \sqsubseteq C_5$$

$$C_9 \sqsubseteq B \sqcup C_8 \sqcup C_6$$

$$C_7 \sqsubseteq C_9$$

$$C_8 \sqsubseteq F$$

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$A \sqsubseteq \exists r.B$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$A \sqsubseteq \exists r.B$$

$$A(x) \rightarrow (\exists r.B)(x)$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$A \sqsubseteq \exists r.B$$

$$A(x) \rightarrow (\exists r.B)(x)$$

$$A \sqsubseteq \forall r.B$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow (\exists r.B)(x)$$
$$A \sqsubseteq \forall r.B \qquad A(x) \wedge r(x, y) \rightarrow B(y)$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$A \sqsubseteq \exists r.B \qquad A(x) \rightarrow (\exists r.B)(x)$$
$$A \sqsubseteq \forall r.B \qquad A(x) \wedge r(x, y) \rightarrow B(y)$$
$$\exists r.A \sqsubseteq B$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$\begin{array}{ll} A \sqsubseteq \exists r.B & A(x) \rightarrow (\exists r.B)(x) \\ A \sqsubseteq \forall r.B & A(x) \wedge r(x, y) \rightarrow B(y) \\ \exists r.A \sqsubseteq B & r(x, y) \wedge A(y) \rightarrow B(x) \end{array}$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$$\begin{array}{ll} A \sqsubseteq \exists r.B & A(x) \rightarrow (\exists r.B)(x) \\ A \sqsubseteq \forall r.B & A(x) \wedge r(x, y) \rightarrow B(y) \\ \exists r.A \sqsubseteq B & r(x, y) \wedge A(y) \rightarrow B(x) \\ \forall r.A \sqsubseteq B & \end{array}$$

Translation into Clauses

a TBox with simplified axioms can now be translated into clauses (written as rules):

$$A_1 \sqcap \dots \sqcap A_n \sqsubseteq B_1 \sqcup \dots \sqcup B_m$$
$$A_1(x) \wedge \dots \wedge A_n(x) \rightarrow B_1(x) \vee \dots \vee B_m(x)$$

If $m = 0$, the rule head contains $\perp(x)$.

$A \sqsubseteq \exists r.B$	$A(x) \rightarrow (\exists r.B)(x)$
$A \sqsubseteq \forall r.B$	$A(x) \wedge r(x, y) \rightarrow B(y)$
$\exists r.A \sqsubseteq B$	$r(x, y) \wedge A(y) \rightarrow B(x)$
$\forall r.A \sqsubseteq B$	$\rightarrow (\exists r. \neg A)(x) \vee B(x)$

Agenda

- Motivation
- Recap: Translation into FOL
- Structural Transformation
- Translation into Clauses
- Summary

Summary

- axioms can be simplified via structural transformation
- simplified axioms can be expressed as rules
- existential quantification in rule heads allowed
- often these rules allow to avoid nondeterminism