Problem 1.1

In the lectures the following example from Description Logics was presented:

\[
\begin{align*}
\mathcal{K}_T : & \quad \text{woman} \sqsubseteq \text{person}, \\
& \quad \text{man} \sqsubseteq \text{person}, \\
& \quad \text{mother} = \text{woman} \sqcap \exists \text{child} : \text{person}, \\
& \quad \text{father} = \text{man} \sqcap \exists \text{child} : \text{person}, \\
& \quad \text{parent} = \text{mother} \sqcup \text{father}, \\
& \quad \text{grandparent} = \text{parent} \sqcap \exists \text{child} : \text{parent}, \\
& \quad \text{father} \_ \text{without} _ \text{son} = \text{father} \sqcap \forall \text{child} : \neg \text{man}
\end{align*}
\]

\[
\begin{align*}
\mathcal{K}_A : & \quad \text{parent} (\text{carl}), \text{parent} (\text{conny}), \\
& \quad \text{child} (\text{conny}, \text{joe}), \text{child} (\text{conny}, \text{carl}), \\
& \quad \text{man} (\text{joe}), \text{man} (\text{carl}), \text{woman} (\text{conny}).
\end{align*}
\]

Are the following consequences valid? **Justify** your answers.

1. \(\mathcal{K}_T \cup \mathcal{K}_A \models \text{grandparent} (\text{conny})\)
2. \(\mathcal{K}_T \cup \mathcal{K}_A \models \text{father} (\text{carl})\)
3. \(\mathcal{K}_T \cup \mathcal{K}_A \models \text{father} \_ \text{without} _ \text{son} (\text{carl})\)

Problem 1.2

Prove that \(F \sqsubseteq G \equiv F \sqcap \neg G = \bot\)

Problem 1.3

Show that \(\text{grandparent} \sqsubseteq_{\mathcal{K}_T} \text{parent}\) by reducing subsumption into concept satisfiability, where \(\mathcal{K}_T\) is the T-Box from Problem 1.1.

Problem 1.4

Is the concept \((\text{father} \sqcap \text{mother})\) satisfiable w.r.t. \(\mathcal{K}_T\) from Problem 1.1?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?
2. Is there a single generalized concept axiom that prevents that a person is female and male?

Problem 1.6
Give an equivalent concept of \((\text{woman} \sqcap \exists \text{child} \cdot \text{person})\) without using the constructors \(\sqcap\) and \(\exists r.C\).

Problem 1.7
Prove the following:
If \((\forall r.C)(a) \in A\), and \(r(a, b) \in A\), then \(A \models C(b)\).

Problem 1.8
Prove the following:
If \((\exists r.C)(a) \in A\), \(A\) is satisfiable, and \(b\) is a Skolem constant, then \(A \cup \{r(a, b), C(b)\}\) is satisfiable as well.

Problem 1.9
Let \(A\) be an ABox. Proof or refute the following claims:

1. If \(A\) contains only elements of the form \(r(a, b)\) where \(r\) is a role name and \(a, b\) are individual names, then \(A\) is satisfiable.

2. If \(A\) contains only elements of the form \(A(x)\) where \(A\) is a concept name and \(a\) is an individual name, then \(A\) is satisfiable.

3. If \(A\) contains only elements of the form \(A(x)\) or \(\neg A(x)\), where \(A\) is an atomic concept name and \(x\) an individual name, then \(A\) is satisfiable.